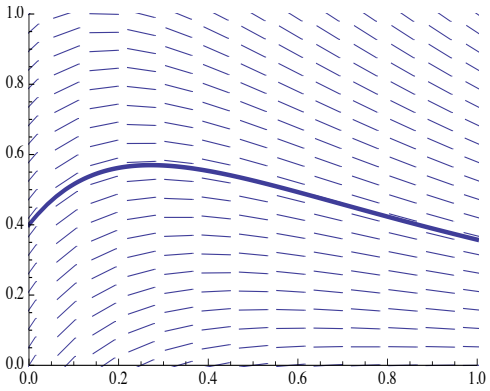
4. [11 points] A restaurant installs a kitchen ventilation system to control the amount of grease in the air due to cooking. The ventilation system reduces the amount of grease in the air by 90 percent every hour. Let $Q(t)$ be the amount in grams of grease in the air t hours after the ventilation is activated. Then Q satisfies the differential equation

$$\frac{dQ}{dt} = 2e^{-5t} - \frac{9}{10}Q,$$

where $2e^{-5t}$ is the rate at which the kitchen produces grease in grams per hour at time t.

a. [2 points] The slope field of the differential equation is shown below. Suppose that the air in the kitchen initially has 0.4 grams of grease. Sketch the solution curve in the slope field.



b. [7 points] Use Euler's method to estimate the values of the solution curve $Q(t)$ through $(0, 0.4)$ for all values of t given in the table below. Show all your work.

Solution:

t	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$Q(t)$	0.4	.9466	.7885	.5757

$Q_{n+1} = Q_n + (2e^{-5t} - \frac{9}{10}Q_n)\Delta t \qquad$ with $\qquad \Delta t = \frac{1}{3}$.

$$\begin{aligned} Q(0) &= 0.4 = Q_0 \\ Q\left(\frac{1}{3}\right) &\approx Q_1 = 0.4 + \left(2 - \frac{9}{10}(0.4)\right)\frac{1}{3} \approx .9466 \\ Q\left(\frac{2}{3}\right) &\approx Q_2 = .9466 + \left(2e^{-\frac{5}{3}} - \frac{9}{10}(.9466)\right)\frac{1}{3} \approx .7885 \\ Q(1) &\approx Q_3 = .7885 + \left(2e^{-\frac{10}{3}} - \frac{9}{10}(.7885)\right)\frac{1}{3} \approx .5757. \end{aligned}$$

c. [2 points] Does your approximation for $Q(1)$ using Euler's method give an overestimate or an underestimate? Justify.

Solution: The estimate is an overestimate since the solution curves are concave down.