

8. [14 points] A coffee shop offers only one hour of free internet access to all its customers. The time  $t$  in hours a customer uses the internet at the coffee shop has a probability density function

$$p(t) = \begin{cases} at\sqrt{1-t^2} & 0 \leq t \leq 1. \\ 0 & \text{otherwise.} \end{cases}$$

where  $a$  is a constant.

- a. [4 points] For what value of  $a$  is  $p(t)$  a probability density function? Find its value without using your calculator.

- b. [4 points] Find the cumulative distribution function  $P(t)$  of  $p(t)$ . Make sure to indicate the value of  $P(t)$  for all values of  $-\infty < t < \infty$ . Your final answer should not contain any integrals.

- c. [3 points] Find the the probability that a customer is still using the internet after 40 minutes (without using your calculator).
- d. [3 points] Find an expression for the mean of this distribution. Use your calculator to compute its value.