1. [8 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] The function $y(t) = \cos(4t)$ is a solution to the differential equation $y'' + 16y = 0$.

[True] False

Solution: $y' = -4\sin(4t)$ and $y'' = -16\cos(4t)$.
Hence $y'' + 16y = -16\cos(4t) + 16\cos(4t) = 0$.

b. [2 points] $\displaystyle\int_1^2 \tan x \; dx$ is an improper integral.

[True] False

Solution: The function $y = \tan(x)$ has a vertical asymptote at $x = \frac{\pi}{2} \approx 1.5708$ which lies inside the interval $[1, 2]$. Hence $\int_1^2 \tan x \; dx$ is improper.

c. [2 points] If $r = f(\theta)$ is a function in polar coordinates with $f''(\theta) > 0$, then its graph in the x-y plane is concave up.

True [False]

Solution: Consider $f(\theta) = e^{-\theta}$, then $f''(\theta) = e^{-\theta} > 0$ for all θ, but its graph is not concave up for all θ.

d. [2 points] The median of the probability density function

$$p(x) = \begin{cases} \frac{1}{x^2} & x \geq 1. \\ 0 & x < 1. \end{cases}$$

is equal to 2.

[True] False

Solution: Since

$$\int_{-\infty}^{2} p(t)dt = \int_1^2 \frac{1}{x^2} dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}.$$

Hence 2 is the median of this distribution.