4. [11 points] The function $P(t)$ models the number of bees (in thousands) in a colony at time t (in years). Suppose the function $P(t)$ satisfies the differential equation

$$\frac{dP}{dt} = 2(1 - 2\sin t)P.$$

The colony initially has 500 bees.

a. [6 points] Use Euler's method, with three steps, to find the approximate number of bees (in thousands) in the farm after one year. Fill in the table with the appropriate values of t and your approximations.

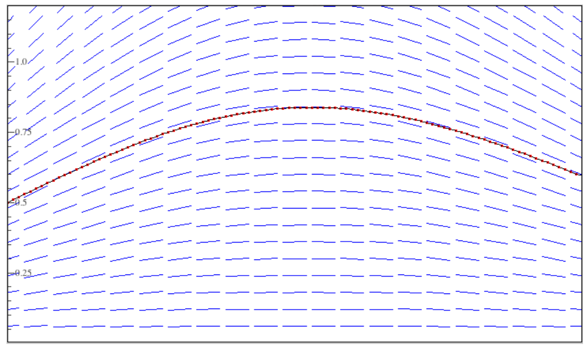
t (in years)	0			1
$P(t)$ (in thousands)				

Solution:

t	P	$\Delta P = 2(1-2\sin t)P\Delta t$
0	0.5	$2(1-2\sin 0)(0.5)(\frac{1}{3}) = \frac{1}{3}$
$\frac{1}{3}$	0.833	$2(1-2\sin\frac{1}{3})(0.833)(\frac{1}{3}) = .191$
$\frac{2}{3}$	1.025	$2(1-2\sin\frac{2}{3})(1.025)(\frac{1}{3}) = -0.161$
1	0.863	

b. [1 point] The slope field of the differential equation $\frac{dP}{dt} = 2(1 - 2\sin t)P$ is shown below. Use it to sketch the graph of $P(t)$, the number of bees (in thousands) in the colony after t years.

Solution:



c. [2 points] Use the differential equation $\frac{dP}{dt} = 2(1 - 2\sin t)P$ to find the exact value of t during the first year at which the number of bees in the colony has a maximum.

Solution: In order to get $\frac{dP}{dt} = 0$ we need either $2(1 - 2\sin t) = 0$ or $P = 0$. Since the number of bees $P(t)$ is never zero in the first year as seen in the slope field above, then at the maximum $2(1 - 2\sin t) = 0$. This occurs when $\sin t = \frac{1}{2}$. During the first year it is at $t = \frac{\pi}{6} \approx .523$ years.

d. [2 points] Does the approximation of $P(1)$ obtained with Euler's method in (a) guarantee an underestimate, an overestimate or neither? Justify without solving the differential equation.

Solution: Euler's method yields an overestimate for $P(1)$ since the function $P(t)$ is concave down (see slope field).