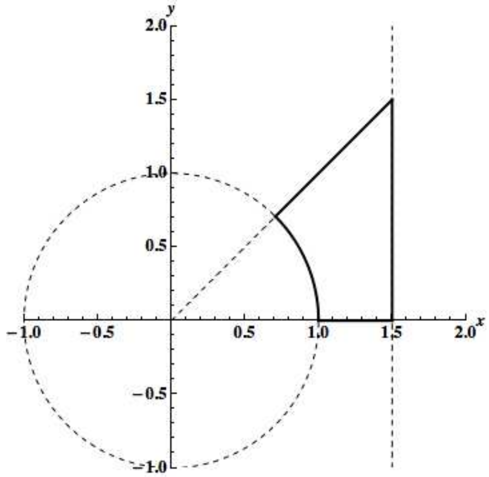
5. [12 points] Solve each of the following problems.

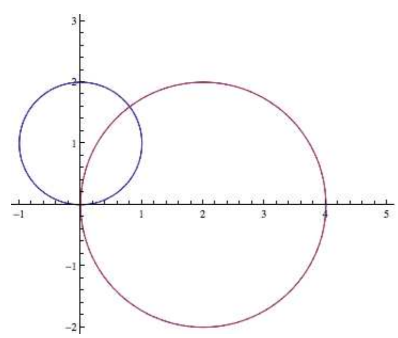
a. [4 points] Give inequalities for r and θ that describe the region shown below in polar coordinates. The region is bounded by the circle $x^2 + y^2 = 1$, the line $y = x$, the x-axis and the vertical line $x = 1.5$.



Solution: The line $y = x$ is the polar line $\theta = \frac{\pi}{4}$, so the limits for θ are $0 \le \theta \le \frac{\pi}{4}$. The values of r range from $r = 1$ on the circle to the line $r\cos\theta = 1.5$, or $r = \frac{1.5}{\cos\theta}$. So the region is

$$\begin{cases} 0 \le \theta \le \frac{\pi}{4} \\ 1 \le r \le \frac{1.5}{\cos\theta} \end{cases}$$

b. [8 points] The functions in polar coordinates $r = 2\sin\theta$ and $r = 4\cos\theta$ represent the circles shown below



Let A be the area of the intersection of these circles. Find an expression involving definite integrals in polar coordinates that computes the value of A. You do not need to evaluate the integrals.

Solution: The curves intersect where $2\sin\theta = 4\cos\theta$, or $\tan\theta = 2$. On the interval $0 \le \theta \le \arctan(2)$, $r = 2\sin\theta$ is the outside curve. On the interval $\arctan(2) \le \theta \le \frac{\pi}{2}$, $r = 4\cos\theta$ is the outside curve. The area is

$$A = \frac{1}{2}\int_0^{\arctan(2)} 4\sin^2\theta d\theta + \frac{1}{2}\int_{\arctan(2)}^{\frac{\pi}{2}} 16\cos^2\theta d\theta.$$