

5. [12 points] Solve each of the following problems.

a. [4 points] Give inequalities for \( r \) and \( \theta \) that describe the region shown below in polar coordinates. The region is bounded by the circle \( x^2 + y^2 = 1 \), the line \( y = x \), the \( x \)-axis and the vertical line \( x = 1.5 \).

\[ \text{Solution:} \quad \text{The line } y = x \text{ is the polar line } \theta = \frac{\pi}{4}, \text{ so the limits for } \theta \text{ are } 0 \leq \theta \leq \frac{\pi}{4}. \text{ The values of } r \text{ range from } r = 1 \text{ on the circle to the line } r \cos \theta = 1.5, \text{ or } r = \frac{1.5}{\cos \theta}. \text{ So the region is} \]

\[ \begin{align*}
0 & \leq \theta \leq \frac{\pi}{4} \\
1 & \leq r \leq \frac{1.5}{\cos \theta}
\end{align*} \]

b. [8 points] The functions in polar coordinates \( r = 2 \sin \theta \) and \( r = 4 \cos \theta \) represent the circles shown below.

\[ \text{Solution:} \quad \text{The curves intersect where } 2 \sin \theta = 4 \cos \theta, \text{ or } \tan \theta = 2. \text{ On the interval } 0 \leq \theta \leq \arctan(2), \text{ } r = 2 \sin \theta \text{ is the outside curve. On the interval } \arctan(2) \leq \theta \leq \frac{\pi}{2}, \text{ } r = 4 \cos \theta \text{ is the outside curve. The area is} \]

\[ A = \frac{1}{2} \int_0^{\arctan(2)} 4 \sin^2 \theta \, d\theta + \frac{1}{2} \int_{\arctan(2)}^{\pi/2} 16 \cos^2 \theta \, d\theta. \]