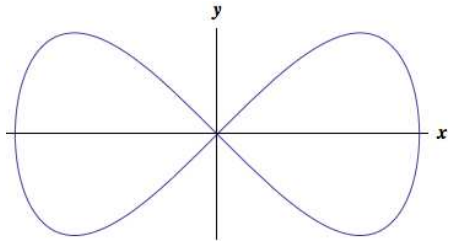
6. [13 points] A particle moves along the path given by the parametric equations

$$x(t) = a\cos t \qquad y(t) = \sin 2t \qquad \text{for } 0 \le t \le 2\pi.$$

where a is a positive constant. The graph of the particle's path in the x-y plane is shown below. In the questions below, show all your work to receive full credit.



a. [2 points] At which values of $0 \le t \le 2\pi$, does the particle pass through the origin?

Solution: $0 = x(t) = a\cos t$: $t = \frac{\pi}{2}, \frac{3\pi}{2}$.
$0 = y(t) = \sin 2t$: $2t = 0, \pi, 2\pi, 3\pi, 4\pi \Rightarrow t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. Particle passes through origin: $t = \frac{\pi}{2}, \frac{3\pi}{2}$.

b. [5 points] For what values of a are the two tangent lines to the curve at the origin perpendicular? Hint: Two lines are perpendicular if the product of their slopes is equal to -1.

Solution: $x'(t) = -a\sin t$, $y'(t) = 2\cos 2t$.

$t = \frac{\pi}{2}$	$t = \frac{3\pi}{2}$
$\frac{dx}{dt} = -a$	$\frac{dx}{dt} = a$
$\frac{dy}{dt} = -2$	$\frac{dy}{dt} = -2$
$\frac{dy}{dx} = \frac{2}{a}$	$\frac{dy}{dx} = -\frac{2}{a}$

$$-1 = \frac{2}{a}\left(-\frac{2}{a}\right) = -\frac{4}{a^2} \Rightarrow a = 2.$$

c. [4 points] At what values of $0 \le t \le 2\pi$, does the curve have horizontal tangents?

Solution:

$$0 = y'(t) = 2\cos 2t \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots, \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

d. [2 points] Find an expression that computes the length of the curve.

Solution:

$$\int_0^{2\pi} \sqrt{a^2\sin^2 t + 4\cos^2 2t}\,dt.$$