1. [10 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] If $r = f(\theta)$ is a polar curve and is concave down, then $f''(\theta) < 0$.

True **[False]**

Solution: The function $f(\theta) = 1$ is concave down for $0 < \theta < \pi$, but $f''(\theta) = 0$.

b. [2 points] Let $y = f(x)$ be a solution of the differential equation $y' = g(x)$ where $g(x)$ is an increasing function. Then the graph of $f(x)$ is concave up.

[True] False

Solution: Since $y'' = g'(x) > 0$, then $f(x)$ is concave up.

c. [2 points] The function $x(t) = e^{-3t} + 2t^2 + \frac{4}{9}$ is a solution to $x'' = 9x - 18t^2$.

[True] False

Solution: $x'' = 9e^{-3t} + 4$ and $9x - 18t^2 = 9(e^{-3t} + 2t^2 + \frac{4}{9}) - 18t^2 = 9e^{-3t} + 4$. Hence $x'' = 9x - 18t^2$.

d. [2 points] If $\int_0^\infty f(x)\,dx$ and $\int_0^\infty g(x)\,dx$ both diverge, then $\int_0^\infty f(x)g(x)dx$ diverges.

True **[False]**

Solution: If $f(x) = g(x) = \frac{1}{x+1}$, then

$$\int_0^\infty f(x)\,dx = \int_0^\infty g(x)\,dx = \int_0^\infty \frac{1}{x+1}\,dx = \lim_{b\to\infty} \ln|x+1||_0^b = \infty \text{ (diverges), but}$$

$$\int_0^\infty f(x)g(x)dx = \int_0^\infty \frac{1}{(1+x)^2}dx = \lim_{b\to\infty} -\frac{1}{1+x}|_0^b = 1 \text{ (converges).}$$

e. [2 points] If $k > 0$ is a constant, then on the interval $a \le t \le b$, the arclength of the parametric curve $x = kf(t)$, $y = kg(t)$ is k times the arclength of $x = f(t), y = g(t)$.

[True] False

Solution: $$\int_a^b \sqrt{\left(\frac{d(kf(t))}{dt}\right)^2 + \left(\frac{d(kg(t))}{dt}\right)^2}dt = k\int_a^b \sqrt{\left(\left(\frac{d(f(t))}{dt}\right)^2 + \left(\frac{d(g(t))}{dt}\right)^2\right)}dt$$