5. [10 points] Consider a group of people who have received a new treatment for pneumonia. Let t be the number of days it takes for a person with pneumonia to fully recover. The probability density function giving the distribution of t is

$$f(t) = \frac{10}{(1+at)^2}, \qquad \text{for } t > 0,$$

for some positive constant a.

a. [2 points] Give a practical interpretation of the quantity $\int_3^{10} f(t)dt$. You do not need to compute the integral.

Solution: The fraction of the people who recovered in between three and ten days after the treatment.

b. [5 points] Find a formula for the cumulative distribution function $F(t)$ of $f(t)$ for $t > 0$. Show all your work. Your answer may include a, but it should not contain any integrals.

Solution: $F(t) = \int_0^t \frac{10}{(1+ax)^2}dx = -\frac{10}{a(1+ax)}\Big|_0^t = \frac{10}{a} - \frac{10}{a(1+at)}$

c. [3 points] Determine the value of a. Show all your work.

Solution: Since $f(t)$ is a probability density function, then $1 = \int_0^\infty \frac{10}{(1+ax)^2}dx$. Hence

$$\int_0^\infty \frac{10}{(1+ax)^2}dx = \lim_{b\to\infty}\int_0^b \frac{10}{(1+ax)^2}dx = \lim_{b\to\infty} \frac{10}{a} - \frac{10}{a(1+ab)} = \frac{10}{a}.$$

Hence $a = 10$.