6. [12 points] Determine the convergence or divergence of the following improper integrals. Justify your answers. Make sure to properly cite any results of convergence or divergence of integrals that you use. If you use the comparison test, be sure to show all your work. Circle your answer.

a. [4 points] $\int_3^\infty \frac{1}{\sqrt[3]{x} + e^{2x}} dx.$ **CONVERGES** DIVERGES

Solution: Since

$$\frac{1}{\sqrt[3]{x} + e^{2x}} \leq \frac{1}{e^{2x}} = e^{-2x}$$

and $\int_3^\infty e^{-2x} dx$ converges then $\int_3^\infty \frac{1}{\sqrt[3]{x} + e^{2x}} dx$ converges.

b. [4 points] $\int_2^\infty \frac{3 + b\sin^2(x^4)}{x^5} dx$, where b is a positive constant.

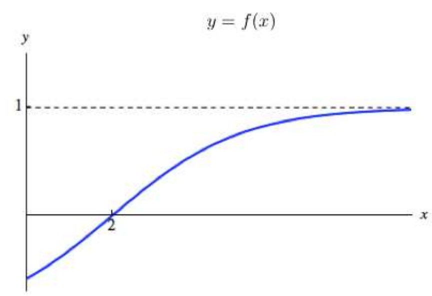
CONVERGES DIVERGES

Solution: Since

$$\frac{3 + b\sin^2(x^4)}{x^5} \leq (3+b)\left(\frac{1}{x^5}\right)$$

and $\int_2^\infty \frac{1}{x^5} dx$ converges ($p > 1$) then $\int_2^\infty \frac{3 + b\sin^2(x^4)}{x^5} dx$ converges.

c. [4 points] Let $f(x)$ be the differentiable function shown below. Note that $f(x)$ has a horizontal asymptote at $y = 1$.



Does $\int_2^\infty \frac{f'(x)}{1 + f(x)} dx$ converge or diverge? Circle your answer. If it converges, find its value.

CONVERGES DIVERGES

Solution:

$$\int_2^\infty \frac{f'(x)}{1+f(x)} dx = \lim_{b\to\infty} \int_2^b \frac{f'(x)}{1+f(x)} dx$$

$$= \lim_{b\to\infty} \ln|1 + f(x)| \Big|_2^b = \lim_{b\to\infty} \ln|1 + f(b)| - \ln|1 + f(2)| = \ln 2.$$