7. [8 points] The population of termites $P(t)$ (in thousands) in a tree grows at a rate $f(P)$, in thousands of termites per day. A pesticide is applied to the tree to eliminate the termites. As a result, the population of termites $P(t)$ satisfies

$$\frac{dP}{dt} = f(P) - 3e^{-\frac{1}{3}t},$$

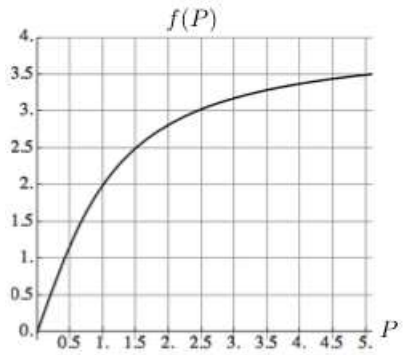
where t is measured in days since the pesticide is applied.

a. [4 points] Use Euler's method with steps of $\Delta t = 0.5$ to estimate the amount of termites in the tree one day after the pesticide is applied. It is estimated that there are 2500 termites in the tree at the time the pesticide is applied ($t = 0$). The graph of $f(P)$ is given below. Show all your computations.

Solution:

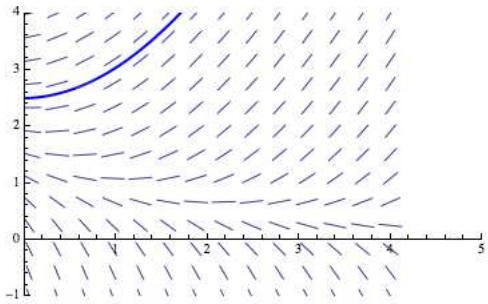
$$\begin{aligned} P(0) &= 2.5 \\ P(0.5) &\approx 2.5 + (0.5)(3 - 3e^{-\frac{1}{3}(0)}) = 2.5 \\ P(1) &\approx 2.5 + (0.5)(3 - 3e^{-\frac{1}{3}(0.5)}) = 2.73. \end{aligned}$$

t	0	0.5	1
$P(t)$	2.5	2.5	2.73



b. [2 points] The slope field of the differential equation satisfied by $P(t)$ is shown below. Sketch the graph of $P(t)$.

Solution:



c. [2 points] Is the estimate obtained in part (**a**) guaranteed to be an overestimate or an underestimate? Justify.

Solution: Underestimate because $P(t)$ is concave up.