**8**. [15 points] Two zombies are chasing Jake down the Diag. Let $J(t)$ be Jake's position, measured in meters along the Diag, as he runs from the zombies. In this problem the time $t$ is measured in seconds.

**a**. [3 points] The velocity of the first zombie is proportional to the difference between its own position, $S(t)$, and Jake's position, with constant of proportionality $k$. Using this fact, write the differential equation satisfied by $S(t)$.

*Solution:*

$$\frac{dS}{dt} = k(S - J(t)).$$

**b**. [2 points] State whether your equation in part $(a)$ is separable. Circle the correct answer.

*Solution:*

The equation is: separable **NOT SEPARABLE**

Note: $J(t)$ is not constant, since Jake is running.

**c**. [9 points] The position of the second zombie at time $t$ is given by the function $Z(t)$ (in meters), and satisfies the differential equation

$$\frac{dZ}{dt} = \alpha \frac{J(t)}{Z},$$

where $\alpha$ is a positive constant. Assuming that $Z(0) = 5$ and that Jake's position is given by $J(t) = 2t + 10$, find a formula for $Z(t)$.

*Solution:* Separating gives:

$$\int Z dZ = \alpha \int 2t + 10 \; dt,$$

and so

$$\tfrac{1}{2}Z^2 = \alpha(t^2 + 10t) + C.$$

Plugging in $Z(0) = 5$, we see that $C = \frac{25}{2}$, so $Z(t)$ is given by:

$$Z(t) = \sqrt{2\alpha t^2 + 20\alpha t + 25}.$$

**d**. [1 point] In the differential equation $\dfrac{dZ}{dt} = \alpha \dfrac{J(t)}{Z}$, what are the units of $\alpha$?

*Solution:* The units are $m/s$.