8. [15 points] Two zombies are chasing Jake down the Diag. Let \( J(t) \) be Jake’s position, measured in meters along the Diag, as he runs from the zombies. In this problem the time \( t \) is measured in seconds.

a. [3 points] The velocity of the first zombie is proportional to the difference between its own position, \( S(t) \), and Jake’s position, with constant of proportionality \( k \). Using this fact, write the differential equation satisfied by \( S(t) \).

\[
\frac{dS}{dt} = k(S - J(t)).
\]

b. [2 points] State whether your equation in part (a) is separable. Circle the correct answer.

Solution:

The equation is: separable NOT SEPARABLE

Note: \( J(t) \) is not constant, since Jake is running.

c. [9 points] The position of the second zombie at time \( t \) is given by the function \( Z(t) \) (in meters), and satisfies the differential equation

\[
\frac{dZ}{dt} = \alpha \frac{J(t)}{Z},
\]

where \( \alpha \) is a positive constant. Assuming that \( Z(0) = 5 \) and that Jake’s position is given by \( J(t) = 2t + 10 \), find a formula for \( Z(t) \).

Solution: Separating gives:

\[
\int ZdZ = \alpha \int 2t + 10 \, dt,
\]

and so

\[
\frac{1}{2}Z^2 = \alpha(t^2 + 10t) + C.
\]

Plugging in \( Z(0) = 5 \), we see that \( C = \frac{25}{2} \), so \( Z(t) \) is given by:

\[
Z(t) = \sqrt{2\alpha t^2 + 20\alpha t + 25}.
\]

d. [1 point] In the differential equation \( \frac{dZ}{dt} = \alpha \frac{J(t)}{Z} \), what are the units of \( \alpha \)?

Solution: The units are \( m/s \).