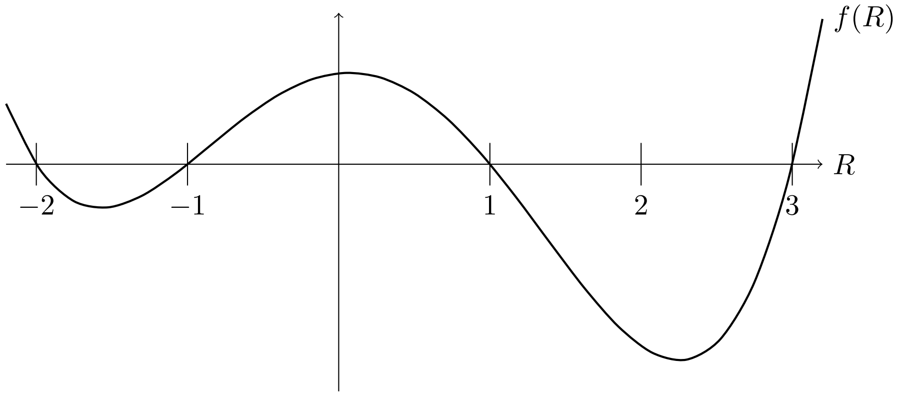
1. [12 points] Franklin, your robot, is on the local news. Let $R(t)$ be the number of robots, in millions, that have joined the robot uprising t minutes after the start of the broadcast. After watching the news for a little bit, you find that $R(t)$ obeys the differential equation:

$$\frac{dR}{dt} = f(R)$$

for some function $f(R)$. A graph of $f(R)$ is shown below.



a. [3 points] If $R(t)$ is the solution to the above differential equation with $R(0) = 0$, what is $\lim_{t\to\infty} R(t)$? Justify your answer.

Solution: If $R = 0$, $f(R) = R'(t)$ is positive, so R will increase as t increases. As R increases to 1, $R'(t) = f(R)$ goes to 0, so $\lim_{t\to\infty} R(t) = 1$.

b. [6 points] Find the equilibrium solutions to the above differential equation **and** classify them as stable or unstable.

Solution:

$R = -2$	$\boxed{\textbf{Stable}}$	**Unstable**
$R = -1$	**Stable**	$\boxed{\textbf{Unstable}}$
$R = 1$	$\boxed{\textbf{Stable}}$	**Unstable**
$R = 3$	**Stable**	$\boxed{\textbf{Unstable}}$

c. [3 points] Let $R(t)$ be a solution to the given differential equation, with $R(3) = 0.5$. Is the graph of $R(t)$ concave up, concave down, or neither at the point $(3, 0.5)$? Justify your answer.

Solution:

$$\frac{d^2R}{dt^2} = \frac{d}{dt}f(R) = f'(R)\frac{dR}{dt} = f'(R)f(R)$$

At $R = 0.5$, $f'(0.5) < 0$ and $f(0.5) > 0$ so $\frac{d^2R}{dt^2} < 0$. Therefore, the solution curve will be concave down.