

7. [8 points] Suppose that $f(x)$ is a differentiable function, defined for $x > 0$, which satisfies the inequalities $0 \leq f(x) \leq \frac{1}{x}$ for $x > 0$. Determine whether the following statements are always, sometimes or never true by circling the appropriate answer. No justification is necessary.

a. [2 points] $\int_1^\infty f(x)dx$ converges.

Always

Sometimes

Never

b. [2 points] $\int_1^\infty (f(x))^2 dx$ converges.

Always

Sometimes

Never

c. [2 points] $\int_0^1 f(x)dx$ converges.

Always

Sometimes

Never

d. [2 points] $\int_1^\infty e^{f(x)} dx$ converges.

Always

Sometimes

Never