2. [8 points] Wild rabbits were introduced to Australia in 1859. The behavior of the rabbit population P in Australia at a time t years after 1859 was modeled by the differential equation

$$P' = P + e^{-t}.$$

a. [4 points] For what value of B is

$$P = 3e^t + Be^{-t}$$

a solution to the differential equation? Be sure to show clearly how you got your answer.

Solution: We can compute that

$$P' = 3e^t - Be^{-t},$$

so if $P = 3e^t - Be^{-t}$ is a solution to the differential equation,

$$3e^t - Be^{-t} = 3e^t + Be^{-t} + e^{-t}.$$

Solving, we see that $B = \frac{-1}{2}$.

b. [4 points] Suppose that the rabbit population in 1859 was 24 rabbits. Historians used Euler's method with $\Delta t = \frac{1}{2}$ to estimate the rabbit population in 1861. Is their answer an overestimate or and underestimate? Give a brief justification of your answer.

Overestimate $\boxed{\textbf{Underestimate}}$

Solution: Taking a derivative of the differential equation, we see that

$$P'' = P' - e^{-t} = (P + e^{-t}) - e^{-t} = P..$$

Since $P(0) = 24 > 0$ and $P' = P + e^{-t} > 0$, P is always positive. Thus $P'' = P > 0$ and so P is concave up, hence Euler's method gives an underestimate.

3. [6 points] Write an explicit expression involving integrals which gives the arc length of **one petal** of the polar rose $r = 3\cos(5\theta)$. Your answer should not contain the letter 'r'. Do not evaluate any integrals.

Solution: The arc length of one petal is

$$\int_{-\pi/10}^{\pi/10} \sqrt{(3\cos(5\theta))^2 + (-15\sin(5\theta))^2}\,d\theta.$$