2. [8 points] Wild rabbits were introduced to Australia in 1859. The behavior of the rabbit population \( P \) in Australia at a time \( t \) years after 1859 was modeled by the differential equation
\[
P' = P + e^{-t}.
\]

a. [4 points] For what value of \( B \) is
\[
P = 3e^t + Be^{-t}
\]
a solution to the differential equation? Be sure to show clearly how you got your answer.

**Solution:** We can compute that
\[
P' = 3e^t - Be^{-t},
\]
so if \( P = 3e^t - Be^{-t} \) is a solution to the differential equation,
\[
3e^t - Be^{-t} = 3e^t + Be^{-t} + e^{-t}.
\]
Solving, we see that \( B = -\frac{1}{2} \).

b. [4 points] Suppose that the rabbit population in 1859 was 24 rabbits. Historians used Euler’s method with \( \Delta t = \frac{1}{2} \) to estimate the rabbit population in 1861. Is their answer an overestimate or and underestimate? Give a brief justification of your answer.

**Overestimate**

**Underestimate**

**Solution:** Taking a derivative of the differential equation, we see that
\[
P'' = P' - e^{-t} = (P + e^{-t}) - e^{-t} = P.
\]
Since \( P(0) = 24 > 0 \) and \( P' = P + e^{-t} > 0 \), \( P \) is always positive. Thus \( P'' = P > 0 \) and so \( P \) is concave up, hence Euler’s method gives an underestimate.

3. [6 points] Write an explicit expression involving integrals which gives the arc length of one petal of the polar rose \( r = 3 \cos(5\theta) \). Your answer should not contain the letter ‘\( r \)’. Do not evaluate any integrals.

**Solution:** The arc length of one petal is
\[
\int_{-\pi/10}^{\pi/10} \sqrt{(3\cos(5\theta))^2 + (-15\sin(5\theta))^2} d\theta.
\]