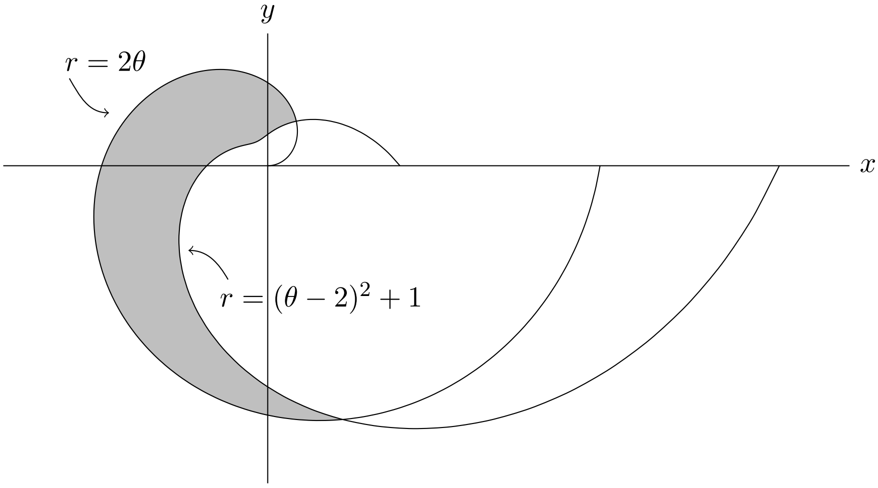
4. [10 points] The visible portion of the strangely-shaped moon of the planet Thethis during its waxing crescent phase is in the shape of the region bounded between the polar curves $r = 2\theta$ and $r = (\theta - 2)^2 + 1$. The region is pictured below. Assume x and y are measured in thousands of miles.



a. [6 points] Write an expression involving integrals which gives the area of the visible portion of this moon. Include the units of the integral in your answer. Do not evaluate any integrals.

Solution: The two curves intersect when $\theta = 1, 5$. Therefore the area of the moon is

$$\left(\int_1^5 \frac{1}{2}(2\theta)^2 d\theta - \int_1^5 \frac{1}{2}((\theta-2)^2+1)^2 d\theta\right) \text{ (thousand miles)}^2$$
$$= \left(\int_1^5 \frac{1}{2}(2\theta)^2 d\theta - \int_1^5 \frac{1}{2}((\theta-2)^2+1)^2 d\theta\right) \text{ million miles}^2.$$

b. [4 points] Find the slope of the tangent line to the polar curve $r = (\theta - 2)^2 + 1$ at $\theta = \pi$.

Solution: Converting to parametric equations, we have

$$x = r\cos\theta = ((\theta-2)^2+1)\cos\theta$$
$$y = r\sin\theta = ((\theta-2)^2+1)\sin\theta$$

Thus

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\left.\frac{dy}{d\theta}\right|_{\theta=\pi}}{\left.\frac{dx}{d\theta}\right|_{\theta=\pi}} = \frac{2(\pi-2)\sin\pi + ((\pi-2)^2+1)\cos\pi}{2(\pi-2)\cos\pi - ((\pi-2)^2+1)\sin\pi} = \frac{(\pi-2)^2+1}{2(\pi-2)}.$$