9. [12 points] Determine if the following integrals converge or diverge. If the integral converges, circle the word "converges". If the integral diverges, circle "diverges". In either case, **you must show all your work and indicate any theorems you use**. You do not need to calculate the value of the integral if it converges.

a. [6 points] $\displaystyle\int_1^\infty \frac{2+\sin x}{\sqrt{x+1}}\,dx$

Converges **Diverges**

Solution: $\dfrac{2+\sin x}{\sqrt{x+1}} \geq \dfrac{1}{\sqrt{2x}}$, so $\displaystyle\int_1^\infty \frac{2+\sin x}{\sqrt{x+1}}\,dx \geq \int_1^\infty \frac{1}{\sqrt{2x}}\,dx$.
But $\displaystyle\int_1^\infty \frac{1}{\sqrt{2x}}\,dx$ diverges by the p-test, $p=\frac{1}{2}\leq 1$, so $\displaystyle\int_1^\infty \frac{2+\sin x}{\sqrt{x+1}}\,dx$ also diverges by the direct comparison test.

b. [6 points] $\displaystyle\int_1^\infty \frac{\theta}{\sqrt{\theta^5+1}}\,d\theta$

Converges Diverges

Solution: $\dfrac{\theta}{\sqrt{\theta^5+1}} \leq \dfrac{\theta}{\sqrt{\theta^5}} = \dfrac{1}{\theta^{3/2}}$, so $\displaystyle\int_1^\infty \frac{\theta}{\sqrt{\theta^5+1}}\,d\theta \leq \int_1^\infty \frac{1}{\theta^{3/2}}\,d\theta$.
But $\displaystyle\int_1^\infty \frac{1}{\theta^{3/2}}\,d\theta$ converges by the p-test, $p=\frac{3}{2}>1$, so $\displaystyle\int_1^\infty \frac{\theta}{\sqrt{\theta^5+1}}\,d\theta$ also converges by the direct comparison test.