

8. [12 points] For each of the following parts, circle the correct answer. Ambiguous answers will receive no credit. You do **not** need to show your work.

a. [3 points] For nonzero constants a and b , the curve $r = \frac{a}{\sin(\theta) + b \cos(\theta)}$ is a line. What is the Cartesian equation of the line?

$$y = -bx + a$$

$$y = ax - b$$

$$y = bx - a$$

$$y = -ax + b$$

$$y = -bx - a$$

b. [3 points] Raymond Green left a bowl of ice cream in a 50°C sauna. Over the first $2 \ln(2)$ hours, the ice cream goes from -10°C to 20°C . Which of the following describes the change in $Q(t)$, the temperature of the ice cream in $^\circ\text{C}$ after t hours?

$$\frac{dQ}{dt} = \frac{Q - 50}{2}$$

$$\frac{dQ}{dt} = 2(50 - Q)$$

$$\frac{dQ}{dt} = \ln(2) \left(25 - \frac{Q}{2} \right)$$

$$\frac{dQ}{dt} = 25 - \frac{Q}{2}$$

$$\frac{dQ}{dt} = -\frac{1}{2}(Q - 25)$$

c. [3 points] Let $\alpha > 0$ be a constant. What is the value of $\lim_{u \rightarrow \infty} \left(\frac{u}{u - \alpha} \right)^{u - \alpha}$?

$$e^\alpha$$

$$1$$

$$\text{DIVERGES}$$

$$e^{1/\alpha}$$

$$\alpha$$

d. [3 points] Consider the differential equation $y' = 1 + \beta xy$, where β is a constant, and let $y(x)$ be a solution satisfying $y(0) = 1$. For which value of β does Euler's method with 2 steps give the estimate $y(4) \approx 0$?

$$-\frac{3}{4}$$

$$-\frac{1}{6}$$

$$-\frac{5}{12}$$

$$-\frac{1}{2}$$

$$-\frac{6}{11}$$