9. [10 points] Suppose that f is function with the following properties:

f is differentiable $\qquad$ $f(x) > 0$ for all x $\qquad$ $\displaystyle\int_1^\infty f(x)\,dx$ converges.

For each of the following parts, determine whether the statement is always, sometimes, or never true by circling the appropriate answer. No justification is needed.

a. [2 points] $\displaystyle\int_{500}^\infty 1000 f(x)\,dx$ converges.

ALWAYS **SOMETIMES** **NEVER**

b. [2 points] $\displaystyle\int_1^\infty (f(x))^{2/3}\,dx$ converges.

ALWAYS **SOMETIMES** **NEVER**

c. [2 points] $\displaystyle\int_1^\infty (f(x))^{3/2}\,dx$ converges.

ALWAYS **SOMETIMES** **NEVER**

d. [2 points] $\displaystyle\int_0^1 f\left(\tfrac{1}{x}\right)\,dx$ converges.

ALWAYS **SOMETIMES** **NEVER**

e. [2 points] $\displaystyle\int_1^\infty \frac{f'(x)}{f(x)}\,dx$ converges. $\qquad$ $\left(\text{ Note: } \dfrac{f'(x)}{f(x)} = \dfrac{d}{dx}\ln(f(x)).\ \right)$

ALWAYS **SOMETIMES** **NEVER**

