11. [12 points] Determine whether the following integrals converge or diverge. If an integral converges, find its exact value (i.e., no decimal approximations) and write it in the blank provided. If it diverges, circle "DIVERGES" and explain why. In any case, **show all your work, indicating any theorems you use, and using proper syntax and notation**.

a. [6 points] $\displaystyle\int_0^\infty 2xe^{-cx}\,dx$, where $c > 0$ is a constant

DIVERGES **CONVERGES TO** $2/c^2$

Solution: For $b \geq 0$ we have

$$\int_0^b xe^{-cx}\,dx = -\frac{xe^{-cx}}{c}\bigg|_0^b + \frac{1}{c}\int_0^b e^{-cx}\,dx = -\frac{be^{-cb}}{c} + \frac{1-e^{-cb}}{c^2},$$

so

$$\int_0^\infty 2xe^{-cx}\,dx = 2\lim_{b\to\infty}\left(-\frac{be^{-cb}}{c} + \frac{1-e^{-cb}}{c^2}\right) = \frac{2}{c^2}.$$

b. [6 points] $\displaystyle\int_0^1 \frac{x}{\sqrt{x^5+x^7}}\,dx$

$\boxed{\textbf{DIVERGES}}$ **CONVERGES TO** ____________

Solution: For $0 < x \leq 1$ we have

$$\frac{x}{\sqrt{x^5+x^7}} \geq \frac{x}{\sqrt{2x^5}} = \frac{1}{\sqrt{2}x^{3/2}}.$$

Since

$$\frac{1}{\sqrt{2}}\int_0^1 \frac{dx}{x^{3/2}}$$

diverges by the p-Test with $p = \frac{3}{2}$, the original integral diverges by comparison.

Alternatively, notice that

$$\lim_{x\to 0^+}\frac{x/\sqrt{x^5+x^7}}{1/x^{3/2}} = 1.$$

Since

$$\int_0^1 \frac{dx}{x^{3/2}}$$

diverges by the p-Test with $p = \frac{3}{2}$, the original integral diverges by the Limit Comparison Test.