2. [5 points] Find constants A and B so that the function $h(w)$, defined for $w > 0$ by

$$h(w) = Aw^3 + \frac{1}{w},$$

is a solution to the differential equation

$$w^2 \frac{dh}{dw} - 3wh + B = 0$$

satisfying $h(1) = \frac{3}{2}$. Show all your work, and write your final answers in the spaces provided.

Solution:

$A =$ 1/2

$B =$ 4

3. [5 points] In a recent presidential election between candidate A and candidate B, Shamcorp's rival company Hawk-I tried fixing the election by changing the votes on some of the ballots. For the last three hours of the election (between 5pm and 8pm), the company gained access to the huge ballot box containing 100 million ballots.

Hawk-I employees removed ballots from the ballot box continuously at a rate of 4 million ballots per hour. Those ballots were removed in proportion to the current ratio in the box. Hawk-I employees then instantly changed the the ballots voting for candidate B to vote for candidate A (leaving any votes for candidate A unchanged) before immediately returning the ballots to the box.

Assume that the ballot box always contains 100 million votes, and that the ballot box only contains votes for candidates A and B.

Write a differential equation that models $a(t)$, the number of ballots voting for candidate A, **in millions**, in the ballot box t hours after Hawk-I began changing votes.

Solution:

$$\frac{da}{dt} = 4 - \frac{a}{25}$$