5. [7 points] The Intern has designed an experiment to stabilize the highly radioactive compound Porcinate. In his experimental setup, the amount $P(t)$ of Porcinate in moles, t hours after the experiment began, should satisfy the differential equation

$$\frac{dP}{dt} - \frac{tP}{\ln(P)} = 0.$$

Use separation of variables to find a solution $P(t)$ satisfying $P(3) = e$.

Solution: We have

$$\int \frac{\ln(P)}{P}\,dP = \int t\,dt,$$

so

$$\frac{(\ln(P))^2}{2} = \frac{t^2}{2} + C$$

for some constant C. The initial condition $P(3) = e$ gives

$$\frac{(\ln(e))^2}{2} = \frac{3^2}{2} + C,$$

so $C = -4$. Hence

$$P(t) = e^{\sqrt{t^2-8}}.$$

6. [5 points] The Intern is also studying a compound called Bovinate. The amount $B(t)$ of Bovinate in moles, t hours after an experiment began, satisfies the differential equation

$$\frac{dB}{dt} = 2B(1-B)(t+B)^2.$$

a. [3 points] List all equilibrium solutions of the differential equation. Indicate whether each is stable or unstable.

Solution: There are two equilibria: $B = 0$ (unstable) and $B = 1$ (stable).

b. [2 points] If the initial amount of Bovinate were 0.5 moles, what would happen to the amount of Bovinate in the long run?

Solution: The amount of Bovinate would approach 1 mole asymptotically from below.