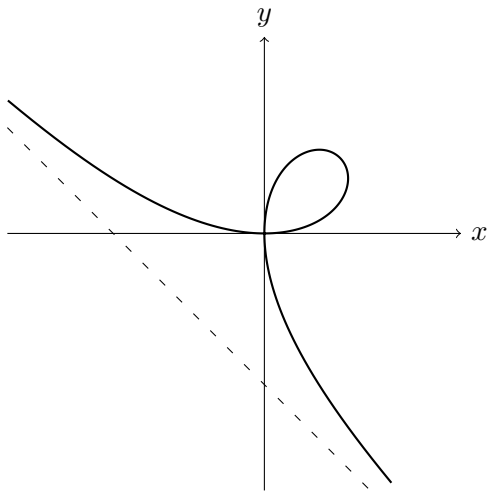
7. [9 points] For $-\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, consider the polar curve

$$r = \frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)}.$$

The curve has an asymptote, the dashed line in the picture, as θ approaches $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$.



a. [4 points] Write down, but do **not** evaluate, an integral that gives the area inside the loop in the first quadrant.

Solution: The area is given by

$$\frac{1}{2}\int_0^{\pi/2} \left(\frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)}\right)^2 d\theta.$$

b. [2 points] Find a formula for the quantity $x + y$ in terms of the variable θ. Write your answer in the space provided.

Solution:

$$x + y = \frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)}(\cos(\theta) + \sin(\theta)) = \sin(2\theta)$$

c. [2 points] Find the limit of $x + y$ as $\theta \to \left(\frac{3\pi}{4}\right)^-$. No justification is needed.

Solution: The specified limit is

$$\lim_{\theta \to (3\pi/4)^-} \sin(2\theta) = \sin\left(\frac{3\pi}{2}\right) = -1.$$

d. [1 point] Write down the Cartesian equation of the asymptote. No justification is needed.

Solution: The asymptote is given by $x + y = -1$.