7. [9 points] For $-\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, consider the polar curve

$$r = \frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)}.$$ 

The curve has an asymptote, the dashed line in the picture, as $\theta$ approaches $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

\[\begin{array}{c}
\text{a. [4 points]} \quad \text{Write down, but do not evaluate, an integral that gives the area inside the loop in the first quadrant.} \\
\text{Solution:} \quad \text{The area is given by} \\
\frac{1}{2} \int_{0}^{\pi/2} \left( \frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)} \right)^2 d\theta.
\end{array}\]

\[\begin{array}{c}
\text{b. [2 points]} \quad \text{Find a formula for the quantity } x + y \text{ in terms of the variable } \theta. \text{ Write your answer in the space provided.} \\
\text{Solution:} \quad x + y = \frac{\sin(2\theta)}{\cos(\theta) + \sin(\theta)}(\cos(\theta) + \sin(\theta)) = \sin(2\theta)
\end{array}\]

\[\begin{array}{c}
\text{c. [2 points]} \quad \text{Find the limit of } x + y \text{ as } \theta \to \left(\frac{3\pi}{4}\right)^-. \text{ No justification is needed.} \\
\text{Solution:} \quad \text{The specified limit is} \\
\lim_{\theta \to (3\pi/4)^-} \sin(2\theta) = \sin \left(\frac{3\pi}{2}\right) = -1.
\end{array}\]

\[\begin{array}{c}
\text{d. [1 point]} \quad \text{Write down the Cartesian equation of the asymptote. No justification is needed.} \\
\text{Solution:} \quad \text{The asymptote is given by } x + y = -1.
\end{array}\]