

8. [12 points] For each of the following parts, circle the correct answer. Ambiguous answers will receive no credit. You do **not** need to show your work.

a. [3 points] For nonzero constants  $a$  and  $b$ , the curve  $r = \frac{a}{\sin(\theta) + b \cos(\theta)}$  is a line. What is the Cartesian equation of the line?

$$\boxed{y = -bx + a}$$

$$y = ax - b$$

$$y = bx - a$$

$$y = -ax + b$$

$$y = -bx - a$$

b. [3 points] Raymond Green left a bowl of ice cream in a  $50^\circ\text{C}$  sauna. Over the first  $2 \ln(2)$  hours, the ice cream goes from  $-10^\circ\text{C}$  to  $20^\circ\text{C}$ . Which of the following describes the change in  $Q(t)$ , the temperature of the ice cream in  $^\circ\text{C}$  after  $t$  hours?

$$\frac{dQ}{dt} = \frac{Q - 50}{2}$$

$$\frac{dQ}{dt} = 2(50 - Q)$$

$$\frac{dQ}{dt} = \ln(2) \left( 25 - \frac{Q}{2} \right)$$

$$\boxed{\frac{dQ}{dt} = 25 - \frac{Q}{2}}$$

$$\frac{dQ}{dt} = -\frac{1}{2}(Q - 25)$$

c. [3 points] Let  $\alpha > 0$  be a constant. What is the value of  $\lim_{u \rightarrow \infty} \left( \frac{u}{u - \alpha} \right)^{u - \alpha}$ ?

$$\boxed{e^\alpha}$$

$$1$$

$$\text{DIVERGES}$$

$$e^{1/\alpha}$$

$$\alpha$$

d. [3 points] Consider the differential equation  $y' = 1 + \beta xy$ , where  $\beta$  is a constant, and let  $y(x)$  be a solution satisfying  $y(0) = 1$ . For which value of  $\beta$  does Euler's method with 2 steps give the estimate  $y(4) \approx 0$ ?

$$-\frac{3}{4}$$

$$-\frac{1}{6}$$

$$\boxed{-\frac{5}{12}}$$

$$-\frac{1}{2}$$

$$-\frac{6}{11}$$