10. [9 points] The sequence $\{\gamma_n\}$ is defined according to the formula

$$\gamma_n = -\ln(n) + \sum_{k=1}^{n} \frac{1}{k}.$$

(You may recall this sequence from team homework 5.) This sequence converges to a positive number γ (which happens to be $\gamma \approx 0.5772156649$).

a. [2 points] Does the sequence $\{\gamma_n^2\}$ converge or diverge? If this sequence converges, compute the value to which this sequence converges, either in terms of the constant γ or with five decimal places of accuracy.

b. [3 points] Does the series $\displaystyle\sum_{n=1}^{\infty} \gamma_n$ converge or diverge? Briefly explain your answer, and if this series converges, compute the value to which the series converges either in terms of the constant γ or with five decimal places of accuracy.

c. [4 points] Let $h_n = \displaystyle\sum_{k=1}^{n} \frac{1}{k}$. Find the value of $\displaystyle\lim_{n\to\infty} \frac{e^{h_n}}{n}$.

You may give your answer either in terms of the constant γ or with five decimal places of accuracy.

Hint: First consider $\displaystyle\lim_{n\to\infty} \ln\left(\frac{e^{h_n}}{n}\right)$.