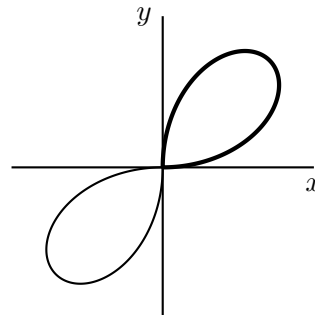


4. [13 points] The orbit of a single electron around the nucleus of an atom is determined by the energy level of that electron and by the other electrons orbiting the nucleus. We can model one electron's orbital in two-dimensions as follows. Suppose that the nucleus of an atom is centered at the origin. Then the (so-called “ $2p_1$ ”) orbital has the shape shown below.

This shape is made up of two regions that we call “lobes”. The outer edge of the lobes are described by the polar equation $r = k \sin(2\theta)$ for some positive constant k . Note that only the relevant portion of the polar curve $r = k \sin(2\theta)$ is shown.

The “top lobe” is the portion in the first quadrant (shown in bold).



- a. [2 points] For what values of θ with $0 \leq \theta \leq 2\pi$ does the polar curve $r = k \sin(2\theta)$ pass through the origin?
- b. [3 points] For what values of θ does the polar curve $r = k \sin(2\theta)$ trace out the “top lobe”? Give your answer as an interval of θ values.
- c. [4 points] Write, but do **not** evaluate, an integral that gives the area of the top lobe.
- d. [4 points] Imagine that an electron lies within the top lobe of this orbital, but is as far away from the origin as possible. What are the polar coordinates of this point of greatest distance from the origin? Your answer may involve the constant k .