

7. [12 points] A bouncy ball is launched up 20 feet from the floor and then begins bouncing. Each time the ball bounces up from floor, it bounces up again to a height that is 60% the height of the previous bounce. (For example, when it bounces up from the floor after falling 20 ft, the ball will bounce up to a height of  $0.6(20) = 12$  feet.)

Consider the following sequences, defined for  $n \geq 1$ :

- Let  $h_n$  be the height, in feet, to which the ball rises when the ball leaves the ground for the  $n$ th time. So  $h_1 = 20$  and  $h_2 = 12$
- Let  $f_n$  be the total distance, in feet, that the ball has traveled (both up and down) when it bounces on the ground for the  $n$ th time. For example,  $f_1 = 40$  and  $f_2 = 40 + 24 = 64$ .

- a. [2 points] Find the values of  $h_3$  and  $f_3$ .

**Answer:**  $h_3 =$  \_\_\_\_\_ and  $f_3 =$  \_\_\_\_\_

- b. [6 points] Find a closed form expression for  $h_n$  and  $f_n$ .

(“Closed form” here means that your answers should not include sigma notation or ellipses  $(\dots)$ . Your answers should also **not** involve recursive formulas.)

**Answer:**  $h_n =$  \_\_\_\_\_ and  $f_n =$  \_\_\_\_\_

- c. [4 points] Decide whether the given sequence or series converges or diverges.

If it diverges, circle “diverges”. If it converges, circle “converges” and write the value to which it converges in the blank.

- i. The sequence  $f_n$

**Converges to** \_\_\_\_\_ **Diverges**

- ii. The series  $\sum_{n=1}^{\infty} h_n$

**Converges to** \_\_\_\_\_ **Diverges**