1. [12 points] Consider the infinite sequences c_n, d_n, j_n and ℓ_n, defined for $n \geq 1$ as follows:

$$c_n = \sum_{k=1}^{n} \frac{(-1)^k}{k!}$$
$$d_n = \arctan(1.1^n)$$
$$j_n = \int_0^{n^3} e^{2x}\,dx$$
$$\ell_n = \sin(x^n) \text{ for some fixed value of } x \text{ satisfying } 0 < x < 1.$$

a. [8 points] Decide whether each of these sequences is bounded, unbounded, always increasing, and/or always decreasing. Record your conclusions by clearly circling the correct descriptions below. Contradictory conclusions will be marked incorrect.

i. The sequence c_n is

$\boxed{\text{bounded}}$ unbounded increasing decreasing

ii. The sequence d_n is

$\boxed{\text{bounded}}$ unbounded $\boxed{\text{increasing}}$ decreasing

iii. The sequence j_n is

bounded $\boxed{\text{unbounded}}$ $\boxed{\text{increasing}}$ decreasing

iv. The sequence ℓ_n is

$\boxed{\text{bounded}}$ unbounded increasing $\boxed{\text{decreasing}}$

b. [4 points]
For parts i and ii below, decide whether the sequence converges or diverges.

- If the sequence converges, circle "converges", find the value to which it converges, and write this value on the answer blank provided.
- If the sequence diverges, circle "diverges".

i. The sequence d_n

$\boxed{\textbf{Converges to } \frac{\pi}{2}}$ **Diverges**

Solution: Note that the geometric sequence 1.1^n diverges to ∞. So since $\lim_{x\to\infty} \arctan(x) = \pi/2$, we have $\lim_{n\to\infty} \arctan(1.1^n) = \pi/2$. That is, the sequence d_n converges to $\pi/2$.

ii. The sequence j_n

Converges to ____________ $\boxed{\textbf{Diverges}}$

Solution: The sequence j_n diverges because it is unbounded or, alternatively, because the improper integral $\int_0^{\infty} e^{2x}\,dx$ diverges (e.g. by direct comparison since $e^{2x} \geq 1$ for $x \geq 0$).