5. [5 points] Consider the improper integral $\displaystyle\int_1^\infty \frac{2}{3x+5e^x}\,dx$.

Note that for $x>0$, we have $\displaystyle\frac{2}{3x+5e^x} < \frac{2}{3x}$ and $\displaystyle\frac{2}{3x+5e^x} < 0.4e^{-x}$.

Use this information together with the (Direct) Comparison Test for Integrals to determine whether $\displaystyle\int_1^\infty \frac{2}{3x+5e^x}\,dx$ converges or diverges.

Write the comparison function you use on the blank below and circle your conclusion for the improper integral. Then briefly explain your reasoning.

Answer: Using (direct) comparison of $\displaystyle\frac{2}{3x+5e^x}$ with the function $0.4e^{-x}$,

the improper integral $\displaystyle\int_1^\infty \frac{2}{3x+5e^x}\,dx$ **Converges** **Diverges**

Briefly explain your reasoning.

Solution: Note that the improper integral $\displaystyle\int_1^\infty e^{-x}\,dx$ is one of the "useful integrals for comparison" from the textbook. This integral is known to converge (exponential decay), so the improper integral $\displaystyle\int_1^\infty 0.4e^{-x}\,dx = 0.4\int_1^\infty e^{-x}\,dx$ also converges. Together with the given inequality $\displaystyle\frac{2}{3x+5e^x} < 0.4e^{-x}$ this implies that the improper integral $\displaystyle\int_1^\infty \frac{2}{3x+5e^x}\,dx$ must also converge by the (Direct) Comparison Test for Improper Integrals.

6. [7 points] Consider the series $\displaystyle\sum_{n=2}^\infty \frac{n^2-n+2}{4n^4-3n^2}$.

Use the **Limit Comparison Test** to determine whether this series converges or diverges.

Circle your answer (either "converges" or "diverges") clearly.

The series $\displaystyle\sum_{n=2}^\infty \frac{n^2-n+2}{4n^4-3n^2}$ **Converges** **Diverges**

Give full evidence to support you answer below. Be sure to clearly state your choice of comparison series, show each step of any computation, and carefully justify your conclusions.

Solution: By considering the exponents in the numerator and the denominator of the general term of this series, we decide to compare this series to $\sum_{n=2}^\infty \frac{1}{4n^2}$.

$$\lim_{n\to\infty} \frac{(n^2-n+2)/(4n^4-3n^2)}{1/4n^2} = \lim_{n\to\infty} \frac{(4n^2)\cdot(n^2-n+2)}{4n^4-3n^2} = 1.$$

Since this limit exists and is non-zero, we can apply the Limit Comparison Test. So the original series and the series $\sum_{n=2}^\infty \frac{1}{4n^2}$ either both converge or both diverge. The series $\sum_{n=2}^\infty \frac{1}{4n^2}$ is 1/4 times a p-series ($p=2$) with the first term ($n=1$) omitted. Omitting a single term and multiplying by a non-zero constant do not affect the convergence of a series, so since the p-series with $p=2$ converges, so too will the series $\sum_{n=2}^\infty \frac{1}{4n^2}$ converge.

By the Limit Comparison Test, we can therefore conclude that the original series $\displaystyle\sum_{n=2}^\infty \frac{n^2-n+2}{4n^4-3n^2}$ must also converge.