8. [9 points] For each of parts **a** through **c** below, circle all of the statements that must be true. Circle "NONE OF THESE" if none of the statements must be true.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Suppose $f(x)$ is a continuous and decreasing function on the interval $[0, 2]$ with $f(0) = 1$ and $f(2) = 0$.

Let a be a constant with $0 < a < 1$. Consider the integral $\displaystyle\int_a^2 \frac{1}{f(x)}\,dx$.

i. This integral is not improper.

ii. This integral converges by direct comparison with the constant function 1.

iii. This integral converges by direct comparison with the function $f(x)$.

iv. This integral converges for some values of a between 0 and 1 but diverges for other values of a between 0 and 1.

v. NONE OF THESE

b. [3 points] Suppose $g(x)$ is a positive and decreasing function that is defined and continuous on the open interval $(5, \infty)$ such that

$\displaystyle\int_{10}^{\infty} g(x)\,dx$ converges and $\displaystyle\int_5^8 g(x)\,dx$ diverges.

i. The series $\displaystyle\sum_{n=20}^{\infty} g(n)$ converges.

ii. The series $\displaystyle\sum_{n=12}^{\infty} \frac{1}{g(n)}$ diverges.

iii. The sequence $c_n = \displaystyle\int_{15}^{n} g(x)\,dx$, $n \geq 15$, converges.

iv. The integral $\displaystyle\int_5^7 g(x)\,dx$ diverges.

v. NONE OF THESE

c. [3 points] Consider the sequence $a_n = \dfrac{1}{\ln(n)}$, $n \geq 2$.

Note: Due to a typo on the original exam (corrected here), all students received full credit for part **c**.

i. $\displaystyle\lim_{n\to\infty} a_n = 0$.

ii. The series $\displaystyle\sum_{n=2}^{\infty} a_n$ converges.

iii. The series $\displaystyle\sum_{n=2}^{\infty} a_n$ diverges.

iv. The series $\displaystyle\sum_{n=2}^{\infty} (-1)^n a_n$ converges.

v. NONE OF THESE