10. [12 points] Provide an example for each of the following. Note that there are examples in each case.

a. [3 points] A sequence $a_n$ that is bounded but not monotonic.

Answer: $a_n =$

b. [3 points] A sequence $b_n$ such that $\sum_{n=1}^{\infty} b_n$ converges but $\sum_{n=1}^{\infty} b_n^2$ diverges.

Answer: $b_n =$

c. [3 points] A sequence $c_n$ and a function $g(x)$ such that $g(n) = c_n$ for all $n \geq 1$, the improper integral $\int_1^{\infty} g(x) \, dx$ diverges, and the series $\sum_{n=1}^{\infty} c_n$ converges.

Note: You may describe your function $g(x)$ by giving either a formula or a well-drawn and clearly labeled graph.

Answer: $c_n =$ and $g(x) =$

d. [3 points] A sequence $d_n$ with $d_n \geq 0$ for $n \geq 1$ such that

$$\lim_{n \to \infty} d_n = 0 \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^n d_n \text{ diverges.}$$

Hint: Consider defining $d_n$ piecewise, with one formula for when $n$ is odd and one for when $n$ is even.

Answer: $d_n =$