7. [12 points] For each of the questions on this page:
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

For parts **a** - **c** below, circle all of the available correct answers, and circle "NONE OF THESE" if none of the available options are correct.

a. [3 points] Let a_n be a sequence of positive numbers, and let $S_n = a_1 + a_2 + \cdots + a_n$. Suppose $\lim\limits_{n \to \infty} \dfrac{S_n}{n^2} = 2$. Which of the following must be true?

   i. The sequence a_n converges.   ii. The sequence S_n diverges.

   iii. The series $\sum\limits_{n=1}^{\infty} a_n$ converges.   iv. The series $\sum\limits_{n=1}^{\infty} S_n$ diverges.

   v. The series $\sum\limits_{n=1}^{\infty} \dfrac{1}{S_n}$ converges.   vi. NONE OF THESE

b. [3 points] Which of the following series are conditionally convergent?

   i. $\sum\limits_{n=1}^{\infty} \left(-\dfrac{1}{3} \right)^n$ ii. $\sum\limits_{n=1}^{\infty} \dfrac{\cos(n)}{n^2}$ iii. $\sum\limits_{n=1}^{\infty} \dfrac{(-2)^n}{n!}$

   iv. $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{\ln(n)}$ v. $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n \, n^3}{4n^3 + 5}$ vi. NONE OF THESE

c. [3 points] Suppose $f(x)$ is a positive, decreasing function on $[0, \infty)$ and suppose $\sum\limits_{n=0}^{\infty} f(n) = 3$. Let $B_n = \int_0^n f(x)\,dx$ for $n \geq 0$, . Which of the following must be true?

   i. $\lim\limits_{n \to \infty} f(n) = 0$ ii. $\lim\limits_{n \to \infty} f(n) = 3$

   iii. $\int_0^{\infty} f(x)\,dx = 3$ iv. $\sum\limits_{n=0}^{\infty} (-1)^n f(n)$ converges

   v. The sequence B_n is bounded and increasing.   vi. NONE OF THESE