

1. [11 points] For each of parts a-d below:

- Find the *exact* value, if possible. Recall that  $x = \sqrt{2}$  is a solution in exact form to the equation  $x^2 = 2$ , but  $x = 1.41421356237$  is not.
- If the given limit or integral either does not exist or diverges, write "DOES NOT EXIST".
- If there is not enough information, write "NOT ENOUGH INFO".
- You do not have to show work, but work shown might be considered for partial credit.

a. [2 points] Suppose  $f(x)$  is a continuous, positive, and decreasing function such that  $\int_2^\infty f(x) dx$  converges. Find  $\lim_{x \rightarrow \infty} f(x)$ .

Answer:  $\lim_{x \rightarrow \infty} f(x) = \underline{0}$

b. [3 points] Find  $\int_0^\infty \frac{1}{x^{0.7}} dx$ .

$$= \lim_{b \rightarrow \infty} \int_0^b x^{-0.7} dx = \lim_{b \rightarrow \infty} \left. \frac{1}{.3} x^{.3} \right|_0^b = \lim_{b \rightarrow \infty} \frac{b^{.3} - 0}{.3} = \infty$$

Answer:  $\int_0^\infty \frac{1}{x^{0.7}} dx = \underline{\text{DOES NOT EXIST}}$

c. [3 points] Suppose  $\mu$  is a real number. Find  $\int_{-\infty}^\infty e^{-(x-\mu)^2/0.0002} dx$ .

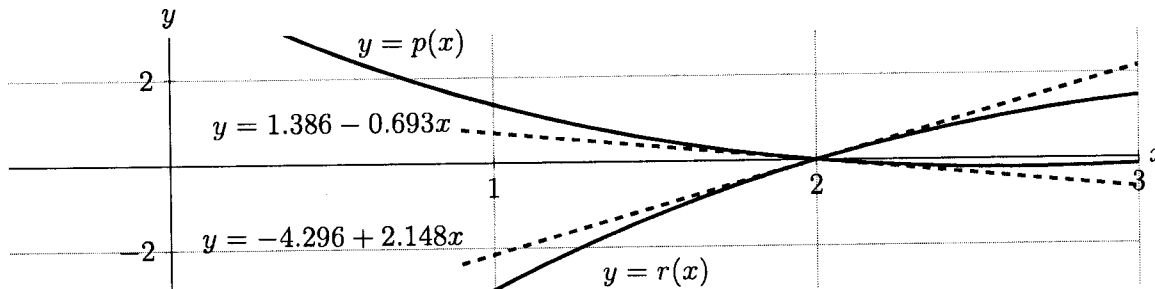
Your answer may involve  $\mu$ .

$$-\frac{(x-\mu)^2}{.0002} = -\frac{(x-\mu)^2}{\frac{2}{10000}} = -\frac{1}{2}(100)^2(x-\mu)^2 = -\frac{1}{2}(100x-100\mu)^2$$

So let  $w = 100x - 100\mu \Rightarrow dw = 100 dx$

Answer:  $\int_{-\infty}^\infty e^{-(x-\mu)^2/0.0002} dx = \int_{-\infty}^\infty e^{-\frac{1}{2}w^2} \cdot \frac{dw}{100} = \boxed{\frac{\sqrt{2\pi}}{100}}$

d. [3 points] The graph below shows two functions  $p(x)$  and  $r(x)$ , as well as their tangent lines at  $x = 2$ .



Find the value of  $\lim_{x \rightarrow 2} \frac{p(x)}{r(x)}$ .

$$\lim_{x \rightarrow 2} \frac{p(x)}{r(x)} = \lim_{x \rightarrow 2} \frac{p'(x)}{r'(x)} = \frac{-0.693}{2.148}$$

by L'Hopital's Rule, since  $\frac{0}{0}$

Answer:  $\lim_{x \rightarrow 2} \frac{p(x)}{r(x)} = \boxed{-\frac{.693}{2.148}}$