10. [12 points] Provide an example for each of the following. Note that there are examples in each case.

a. [3 points] A sequence a_n that is bounded but not monotonic.

Answer: $a_n =$ $(-1)^n$

b. [3 points] A sequence b_n such that $\sum_{n=1}^{\infty} b_n$ converges but $\sum_{n=1}^{\infty} b_n^2$ diverges.

Answer: $b_n =$ $\dfrac{(-1)^n}{\sqrt{n}}$

c. [3 points] A sequence c_n and a function $g(x)$ such that $g(n) = c_n$ for all $n \geq 1$, the improper integral $\int_1^{\infty} g(x)\,dx$ diverges, and the series $\sum_{n=1}^{\infty} c_n$ converges.

Note: You may describe your function $g(x)$ by giving either a formula or a well-drawn and clearly labeled graph.

Must violate one of the conditions of the integral test, so either not positive or not decreasing.

Answer: $c_n =$ 0 and $g(x) =$ $\sin(\pi x)$

d. [3 points] A sequence d_n with $d_n \geq 0$ for $n \geq 1$ such that

$$\lim_{n\to\infty} d_n = 0 \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^n d_n \text{ diverges.}$$

Hint: Consider defining d_n piecewise, with one formula for when n is odd and one for when n is even.

Since AST doesn't apply even though terms alternate and $\to 0$, must be the case that terms don't decrease in magnitude.

Answer: $d_n = \begin{cases} 2/n & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

So $\sum_{1}^{\infty} (-1)^n d_n = -0 + \frac{2}{2} - 0 + \frac{2}{4} - 0 + \frac{2}{6} - \cdots = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$