2. [9 points] Note: "Closed form" here means that the expression should NOT include sigma notation or ellipses (...) and should NOT be recursive.

Michel is studying how the mass of a certain collection of bacterial cells behaves in the presence of a parasite. He notices that from noon to midnight of each day, the parasite eats 60% of the mass of the bacterial cells. Then the parasite sleeps until noon the next day. While the parasite sleeps, the remaining 40% of the collection of bacterial cells doubles in mass.

At noon on the first day, the mass of the collection of bacterial cells is 100 grams.

a. [3 points] Let $X_n$ be the mass, in grams, of bacterial cells present at noon on day $n$.
   Note that $X_1 = 100$. Calculate $X_2$ and $X_3$, and find a closed form expression for $X_n$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$X_n$</th>
<th>Unseen</th>
<th>After regrowth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>(.4)(100)</td>
<td>2 (.4)(100) = 80 = (.8)(100)</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>(.4)(80)</td>
<td>2 (.4)(80) = 64 = (.8)^2(100)</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer: $X_2 = \frac{80}{g}$ and $X_3 = \frac{64}{g}$

Answer: $X_n = \frac{100(.8)^{n-1}}{g}$

b. [4 points] Let $K_n$ be the total mass, in grams, of bacterial cells that the parasite has consumed in the first $n$ days. For example, on day 1 the parasite consumes 60% of 100 grams, which is 60 grams, so $K_1 = 60$. Calculate $K_2$ and $K_3$, and find a closed form expression for $K_n$.

\[
\begin{array}{c|c|c|c|c}
 n & K_n & \text{Amount eaten on day } i & i & \text{Amount eaten on day } i \\
 1 & 60 & 60 & \frac{1}{60} & 60 \\
 2 & 108 & 60 + (.8)(60) & \frac{1}{60} + \frac{1}{60}(.8) & 60 + 60(.8) \\
 3 & 146.4 & 108 + (.6)(60) & \frac{1}{60} + \frac{1}{60}(.8) + \frac{1}{60}(.8)^2 & 60 + 60(.8) + 60(.8)^2 \\
\end{array}
\]

Answer: $K_2 = \frac{108}{g}$ and $K_3 = \frac{146.4}{g}$

Answer: $K_n = \frac{300 (1 - (.8)^n)}{g}$

c. [2 points] If this continued forever, how many grams of bacterial cells would the parasite eventually eat?

\[
\lim_{n \to \infty} 300 (1 - (.8)^n) = 300
\]

Answer: Mass = $\frac{300}{g}$