2. [9 points] Note: *"Closed form" here means that the expression should* NOT *include sigma notation or ellipses (...) and should* NOT *be recursive.*

Michel is studying how the mass of a certain collection of bacterial cells behaves in the presence of a parasite. He notices that from noon to midnight of each day, the parasite eats 60% of the mass of the bacterial cells. Then the parasite sleeps until noon the next day. While the parasite sleeps, the remaining 40% of the collection of bacterial cells doubles in mass.

At noon on the first day, the mass of the collection of bacterial cells is 100 grams.

a. [3 points] Let X_n be the mass, in grams, of bacterial cells present at noon on day n. Note that $X_1 = 100$. Calculate X_2 and X_3, and find a closed form expression for X_n.

n	X_n	Uneaten	After regrowth
1	100	$(.4)(100)$	$2(.4)(100) = 80 = (.8)(100)$
2	80	$(.4)(80)$	$2(.4)(80) = 64 = (.8)^2(100)$
3	64		

Answer: $X_2 =$ 80 g and $X_3 =$ 64 g

Answer: $X_n =$ $100(.8)^{n-1}$ g

b. [4 points] Let K_n be the total mass, in grams, of bacterial cells that the parasite has consumed in the first n days. For example, on day 1 the parasite consumes 60% of 100 grams, which is 60 grams, so $K_1 = 60$. Calculate K_2 and K_3, and find a closed form expression for K_n.

n	K_n
1	60
2	$60 + (.6)(80) = 108$
3	$108 + (.6)(64) = 146.4$

Amount eaten on day i is $(.6)X_i = (.6)(100)(.8)^{i-1} = 60(.8)^{i-1}$

so $K_n = \sum_{i=1}^{n} 60(.8)^{i-1} = 60\left(1 + (.8) + (.8)^2 + \cdots + (.8)^{n-1}\right) = 60\dfrac{1-(.8)^n}{1-.8}$

$= \dfrac{60}{.2}\left(1-(.8)^n\right) = 300\left(1-(.8)^n\right)$

Answer: $K_2 =$ 108 g and $K_3 =$ 146.4 g

Answer: $K_n =$ $300\left(1-(.8)^n\right)$ g

c. [2 points] If this continued forever, how many grams of bacterial cells would the parasite eventually eat?

$$\lim_{n\to\infty} 300\left(1-(.8)^n\right) = 300$$

Answer: Mass = 300 g