4. [8 points] Determine whether the following series converge or diverge.
Fully justify your answer. Show all work and include any convergence tests used.

a. [4 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sin\left(\frac{1}{n}\right)}$

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

Justification:

As $n \to \infty$, $\frac{1}{n} \to 0$, so $\sin\left(\frac{1}{n}\right) \to \sin(0) = 0$

which means $\frac{1}{\sin\left(\frac{1}{n}\right)}$ does not approach 0.

Since the terms of the series do not approach 0,

the series diverges

b. [4 points] $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n^2+3^n}$

Circle one: $\boxed{\textbf{Converges}}$ **Diverges**

Justification:

Ratio test:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{2^{n+1}}{(n+1)^2+3^{n+1}} \cdot \frac{n^2+3^n}{2^n}\right|$$

$$= \lim_{n\to\infty} \frac{2^{n+1}}{2^n} \cdot \frac{n^2+3^n}{(n+1)^2+3^{n+1}} = 2\lim_{n\to\infty} \frac{n^2+3^n}{(n+1)^2+3^{n+1}}.$$

The exponentials dominate the polynomials, so

that's $2\lim_{n\to\infty} \frac{3^n}{3^{n+1}} = \frac{2}{3}$, which is less than 1.

So converges by the ratio test.