5. [10 points] You are at a bus stop waiting for a bus to arrive. The cumulative distribution function for the time, in minutes, a passenger will wait for the next bus to arrive is given by

$$P(t) = \begin{cases} 0 & t \le 0 \\ 1 - e^{-0.05t} & t > 0. \end{cases}$$

a. [3 points] What is the median amount of time that a passenger has to wait for a bus to arrive? Provide an **exact** answer. Remember to show all your work.

If T is the median time, then

$\frac{1}{2} = \text{Prob}(\text{wait} < T) = P(T) = 1 - e^{-.05T} \Rightarrow e^{-.05T} = \frac{1}{2}$

Answer: Median = $\boxed{\dfrac{\ln(.5)}{-.05}} = 20\ln 2$

You decide that you are going to take the 2nd bus that arrives. It can be shown that the number of minutes a passenger has to wait for 2 buses to arrive has probability density function

$$q(t) = \begin{cases} 0 & t \le 0 \\ Cte^{-0.05t} & t > 0 \end{cases}$$

for some constant C.

b. [5 points] Find the value of C. **Show all your work using correct notation.** Any evaluation of integrals must be done **without a calculator.**

Let $w = .05t \Rightarrow t = 20w$
$dt = 20dw$
$t = 0 \Rightarrow w = 0$
$t \to \infty \Rightarrow w \to \infty$

$1 = \int_{-\infty}^{\infty} q(t)\,dt = \int_0^{\infty} Cte^{-.05t}\,dt$

$= C\int_0^{\infty} (20w)e^{-w}(20dw) = 400C\int_0^{\infty} we^{-w}\,dw$

Let $u = w$ $v' = e^{-w}$
$u' = 1$ $v = -e^{-w}$

$= 400C \lim_{b\to\infty} \left[-we^{-w}\Big|_0^b - \int_0^b -e^{-w}\,dw\right]$

$= 400C \lim_{b\to\infty} \left[\frac{-b}{e^b} - e^{-w}\Big|_0^b\right] = 400C \lim_{b\to\infty}\left[\cancel{\frac{-b}{e^b}}^{\,0} \cancel{-1} + e^{-0}\right]$

$= 400C.$

Answer: $C = 1/400 = .0025$

c. [2 points] Write an expression (possibly involving one or more integrals) for the mean number of minutes it takes for 2 buses to arrive. (You do not need to evaluate your expression.)

$\int_{-\infty}^{\infty} t\,q(t)\,dt$

Answer: Mean = $\frac{1}{400}\int_0^{\infty} t^2 e^{-.05t}\,dt$