7. [12 points] For each of the questions on this page:
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

For parts **a - c** below, circle all of the available correct answers, and circle "NONE OF THESE" if none of the available options are correct.

a. [3 points] Let a_n be a sequence of positive numbers, and let $S_n = a_1 + a_2 + \cdots + a_n$. Suppose $\lim_{n\to\infty} \frac{S_n}{n^2} = 2$. Which of the following must be true?

Suppose $a_n = 4n-2$. Then the sequence a_n and the series $\sum a_n$ both diverge, even though $S_n = 2+6+10+\cdots+(4n-2) = 2n^2$. So that's a counterexample for (i) and (ii)

i. The sequence a_n converges.

$\boxed{\text{ii. The sequence } S_n \text{ diverges.}}$

$S_n \approx 2n^2$ for large n

iii. The series $\sum_{n=1}^{\infty} a_n$ converges.

$\boxed{\text{iv. The series } \sum_{n=1}^{\infty} S_n \text{ diverges.}}$

$S_n \geq a_1 > 0$ so terms don't go to 0

$\boxed{\text{v. The series } \sum_{n=1}^{\infty} \frac{1}{S_n} \text{ converges.}}$

vi. NONE OF THESE

Limit compare with $\sum \frac{1}{n^2}$

b. [3 points] Which of the following series are conditionally convergent?

i. $\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n$

ii. $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$

iii. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

$\boxed{\text{iv. } \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}}$

v. $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{4n^3+5}$

vi. NONE OF THESE

(i), (ii), (iii) are absolutely convergent
(v) is divergent

c. [3 points] Suppose $f(x)$ is a positive, decreasing function on $[0, \infty)$ and suppose $\sum_{n=0}^{\infty} f(n) = 3$. Let $B_n = \int_0^n f(x)\,dx$ for $n \geq 0$, . Which of the following must be true?

$\boxed{\text{i. } \lim_{n\to\infty} f(n) = 0}$

ii. $\lim_{n\to\infty} f(n) = 3$

iii. $\int_0^{\infty} f(x)\,dx = 3$

$\boxed{\text{iv. } \sum_{n=0}^{\infty} (-1)^n f(n) \text{ converges}}$ by AST

$\boxed{\text{v. The sequence } B_n \text{ is bounded and increasing.}}$

vi. NONE OF THESE

increasing because $f(x) > 0$, bounded because $\int_0^{\infty} f(x)dx$ converges by integral test