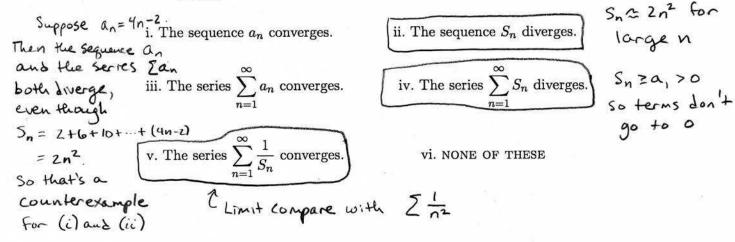
7. [12 points] For each of the questions on this page:

You must circle at least one choice to receive any credit.

No credit will be awarded for unclear markings. No justification is necessary.

For parts a-c below, circle <u>all</u> of the available correct answers, and circle "NONE OF THESE" if none of the available options are correct.

a. [3 points] Let  $a_n$  be a sequence of positive numbers, and let  $S_n = a_1 + a_2 + \cdots + a_n$ . Suppose  $\lim_{n \to \infty} \frac{S_n}{n^2} = 2$ . Which of the following <u>must</u> be true?



b. [3 points] Which of the following series are conditionally convergent?

i. 
$$\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n$$
ii. 
$$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$
iii. 
$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$$
iv. 
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$
v. 
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{4n^3 + 5}$$
vi. NONE OF THESE
(i), (ii), (iii) are absolutely convergent
(v) is divergent

c. [3 points] Suppose f(x) is a positive, decreasing function on  $[0, \infty)$  and suppose  $\sum_{n=0}^{\infty} f(n) = 3. \text{ Let } B_n = \int_0^n f(x) \, dx \text{ for } n \ge 0, \text{ . Which of the following } \underline{\text{must}} \text{ be true?}$ 

ii. 
$$\lim_{n\to\infty} f(n) = 0$$
 iii.  $\lim_{n\to\infty} f(n) = 3$ 
iii.  $\int_0^\infty f(x) \, dx = 3$  iv.  $\sum_{n=0}^\infty (-1)^n f(n)$  converges by  $A > T$ 
v. The sequence  $B_n$  is bounded and increasing. vi. NONE OF THESE follows because  $f(x) > 0$ , bounded because  $f(x) > 0$ , bounded because  $f(x) > 0$  converges by integral test