10. [9 points]

a. [3 points] Let $\sum_{n=1}^{\infty} a_n$ be a series, and let $S_j = a_1 + a_2 + \cdots + a_j$ be the partial sum of the first j terms of the series. If $S_j = \dfrac{4}{1+\frac{1}{j}}$, which of the following statements **must** be true? Circle all correct answers.

i. $\sum_{n=1}^{\infty} a_n$ diverges

ii. $\sum_{n=1}^{\infty} a_n$ converges

iii. $\sum_{n=1}^{\infty} a_n = 4$

iv. the sequence a_n converges

v. the sequence S_j converges to 4

vi. the sequence S_j diverges

vii. $a_n = \dfrac{4}{1+\frac{1}{n}} - \dfrac{4}{1+\frac{1}{n-1}}$ for $n \geq 2$

viii. NONE OF THESE

b. [3 points] Let $h(x)$ be a positive, continuous, decreasing function such that $\int_1^{\infty} h(x)\,dx = 32$, and let $b_n = h(n)$. Which of the following **must** be true? Circle all correct answers.

i. $\sum_{n=1}^{\infty} b_n$ diverges

ii. $\sum_{n=1}^{\infty} b_n$ converges

iii. $\sum_{n=1}^{\infty} b_n = 32$

iv. $\sum_{n=1}^{\infty} (-1)^n b_n$ converges

v. NONE OF THESE

c. [3 points] The force required to compress a spring by a distance of x meters from its equilibrium position is given by $F = kx$, for some constant k, measured in newtons/meter. Let $p(k)$ be the probability density function of the value of k of a batch of springs.

Which of the following represents the probability that the force for compressing a spring in this batch 0.1 m from its equilibrium position is between 0.4 and 0.6 newtons? Circle the one best answer.

i. $\int_4^6 p(k)\,dk$

ii. $p(6) - p(4)$

iii. $\int_{0.4}^{0.6} p(k)\,dk$

iv. $p(0.6) - p(0.4)$

v. $p(120) - p(80)$

vi. NONE OF THESE