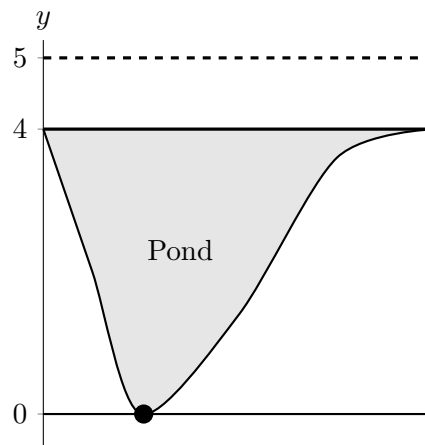


9. [9 points]

A small pond has murky water, and needs to be completely drained.

- A side view of the pond looks like the diagram at right.
- $y$  measures the distance, in meters, above the bottom of the pond.
- The surface of the pond is at  $y = 4$ .
- The water must be pumped to a height **1 meter above the surface**.
- The cross-sections perpendicular to the  $y$ -axis are **circles**.
- The **radius** of the circular cross-section  $y$  meters above the bottom of the pond is  $r(y)$  meters.



- The **density** of the murky water varies with  $y$ , and is given by  $Q(y)$  kg/m<sup>3</sup>.
- Note that the domain for both  $r$  and  $Q$  is  $[0, 4]$ .
- You may assume that acceleration due to gravity is  $g = 9.8$  m/s<sup>2</sup>.

Note that your answers below may include  $r(y)$  and  $Q(y)$ .

- a. [3 points] Write an expression that gives the approximate mass, in kilograms, of a slice of the murky water that is  $\Delta y$  m thick and at a height of  $y$  meters. Your expression should not involve any integrals.

**Answer:** Mass of slice  $\approx$  \_\_\_\_\_

- b. [3 points] Write an expression in terms of  $y$  that approximates the work, in joules, done in pumping a horizontal slice of murky water of thickness  $\Delta y$  at a height of  $y$  meters to 1 meter above the surface of the pond. Your expression should not involve any integrals.

**Answer:** Work  $\approx$  \_\_\_\_\_

- c. [3 points] Write an expression involving one or more integrals that gives the total work, in joules, to completely drain the pond by pumping all the water to 1 meter above the pond.

**Answer:** \_\_\_\_\_