11. [15 points] Let $a_n$, $b_n$, and $c_n$ be sequences such that

- $a_n > 0$ for all $n$,
- $a_n < b_n < n$ for all $n$,
- $\sum_{n=1}^{\infty} a_n$ diverges,
- $3 < c_n < 4$ for all $n$.

Determine whether each of the following must diverge, must converge, or if there is not enough information to decide.

If you say that a series must converge or diverge:

- name or state a test that can be used to justify your claim.
- If you use the direct or limit comparison test, also name the comparison series you would use.

You do not need to justify the convergence or divergence of your comparison series.

a. [3 points] $\sum_{n=1}^{\infty} b_n$

Answer (circle one): Diverges Converges or Not enough info

Solution: Comparison test, $\sum_{n=1}^{\infty} a_n$.

b. [3 points] $\sum_{n=1}^{\infty} \frac{1}{b_n}$

Answer (circle one): Diverges Converges or Not enough info

Solution: Comparison test, $\sum_{n=1}^{\infty} \frac{1}{n}$.

c. [3 points] $\sum_{n=1}^{\infty} c_n$

Answer (circle one): Diverges Converges or Not enough info

Solution: The $n^{th}$ term test for divergence, or comparison test with $\sum_{n=1}^{\infty} 3$.

d. [3 points] $\sum_{n=1}^{\infty} \frac{1}{(c_n)^n}$

Answer (circle one): Diverges Converges or Not enough info

Solution: Comparison test, $\sum_{n=1}^{\infty} \frac{1}{3^n}$.

e. [3 points] $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{a_n}$

Answer (circle one): Diverges Converges or Not enough info