11. [15 points] Let a_n, b_n, and c_n be sequences such that

- $a_n > 0$ for all n,
- $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges,

- $a_n < b_n < n$ for all n,

- $3 < c_n < 4$ for all n.

Determine whether each of the following must diverge, must converge, or if there is not enough information to decide.

If you say that a series must converge or diverge:

- **name or state a test** that can be used to justify your claim.
- If you use the direct or limit comparison test, also **name the comparison series** you would use.

  You do *not* need to justify the convergence or divergence of your comparison series.

a. [3 points] $\displaystyle\sum_{n=1}^{\infty} b_n$

Answer *(circle one)*: ☐ DIVERGES   CONVERGES   or   NOT ENOUGH INFO

Solution: Comparison test, $\displaystyle\sum_{n=1}^{\infty} a_n$.

b. [3 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{b_n}$

Answer *(circle one)*: ☐ DIVERGES   CONVERGES   or   NOT ENOUGH INFO

Solution: Comparison test, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

c. [3 points] $\displaystyle\sum_{n=1}^{\infty} c_n$

Answer *(circle one)*: ☐ DIVERGES   CONVERGES   or   NOT ENOUGH INFO

Solution: The nth term test for divergence, or comparison test with $\displaystyle\sum_{n=1}^{\infty} 3$.

d. [3 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(c_n)^n}$

Answer *(circle one)*: DIVERGES   ☐ CONVERGES   or   NOT ENOUGH INFO

Solution: Comparison test, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$.

e. [3 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{a_n}$

Answer *(circle one)*: DIVERGES   CONVERGES   or   ☐ NOT ENOUGH INFO