

2. [9 points] Note: “Closed form” here means that the expression should NOT include sigma notation or ellipses (...) and should NOT be recursive.

In the live-action series adaptation of *Derivative Girl*, Derivative Girl can lift  $D_0 = 1000$  kg, and she can make as many copies of herself as she wants. The first copy can lift  $1/3$  the amount Derivative Girl can lift, and the  $n$ th copy can lift  $1/3$  the amount the  $(n - 1)$ st copy can lift.

- a. [3 points] Let  $D_n$  be the amount of mass, in kg, that the  $n$ th copy of Derivative Girl can lift. Calculate  $D_1$  and  $D_2$ , and give a closed-form expression for  $D_n$  in terms of  $n$ :

$$D_1 = \underline{1000(\frac{1}{3}) \approx 333.33}$$

$$D_2 = \underline{1000(\frac{1}{3^2}) \approx 111.11}$$

$$D_n = \underline{1000(\frac{1}{3^n})}$$

- b. [4 points] Let  $G_n$  be the amount of mass, in kg, that Derivative Girl and the first  $n$  copies can lift together. Calculate  $G_1$  and  $G_2$ , and give a closed-form expression for  $G_n$ :

$$G_1 = \underline{1000 + 1000(\frac{1}{3}) \approx 1333.33}$$

$$G_2 = \underline{1000 + 1000(\frac{1}{3}) + 1000(\frac{1}{9}) \approx 1444.44}$$

$$G_n = \underline{\frac{1000(1 - (\frac{1}{3})^{n+1})}{1 - 1/3}}$$

- c. [2 points] If Derivative Girl could make infinitely many copies, what is the largest amount, in kg, that Derivative Girl and her copies could lift together? Your answer should be a closed-form expression.

Answer:  $\underline{1500}$