2. [9 points] Note: “Closed form” here means that the expression should NOT include sigma notation or ellipses (…) and should NOT be recursive.

In the live-action series adaptation of Derivative Girl, Derivative Girl can lift $D_0 = 1000$ kg, and she can make as many copies of herself as she wants. The first copy can lift $1/3$ the amount Derivative Girl can lift, and the $n$th copy can lift $1/3$ the amount the $(n - 1)$st copy can lift.

a. [3 points] Let $D_n$ be the amount of mass, in kg, that the $n$th copy of Derivative Girl can lift. Calculate $D_1$ and $D_2$, and give a closed-form expression for $D_n$ in terms of $n$:

$$D_1 = 1000 \left( \frac{1}{3} \right) \approx 333.33$$

$$D_2 = 1000 \left( \frac{1}{3^2} \right) \approx 111.11$$

$$D_n = 1000 \left( \frac{1}{3^n} \right)$$

b. [4 points] Let $G_n$ be the amount of mass, in kg, that Derivative Girl and the first $n$ copies can lift together. Calculate $G_1$ and $G_2$, and give a closed-form expression for $G_n$:

$$G_1 = 1000 + 1000 \left( \frac{1}{3} \right) \approx 1333.33$$

$$G_2 = 1000 + 1000 \left( \frac{1}{3} \right) + 1000 \left( \frac{1}{3^2} \right) \approx 1444.44$$

$$G_n = \frac{1000 \left( 1 - \left( \frac{1}{3} \right)^{n+1} \right)}{1 - 1/3}$$

C. [2 points] If Derivative Girl could make infinitely many copies, what is the largest amount, in kg, that Derivative Girl and her copies could lift together? Your answer should be a closed-form expression.

Answer: 1500