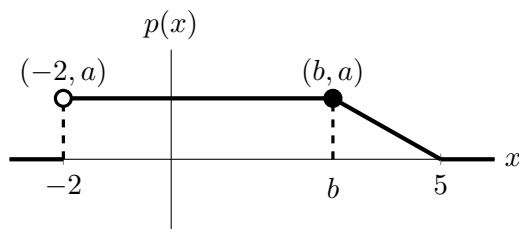


3. [8 points] Anya is playing a game. Each turn, Anya's score can change by  $x$  points, where  $x$  is a real number between  $-2$  and  $5$ . That is, her score can go up by as much as 5 points or down by as much as 2 points. The probability density function for the change in her score each turn is given by the piecewise-linear function  $p(x)$  graphed below:



Here,  $a$  and  $b$  are positive constants. Do not assume the graph shown is to scale.

- a. [6 points] The median amount of points Anya can score each turn is  $b - 1$ . Find the values of  $a$  and  $b$ .

*Solution:* This is a pdf, so

$$\begin{aligned} 1 &= (b - (-2))a + \frac{1}{2}(5 - b)a \\ &= a\left(\frac{1}{2}b + \frac{9}{2}\right) \end{aligned}$$

So  $a = \frac{2}{b+9}$ . The median is  $b - 1$ , so

$$\int_{-2}^{b-1} p(x) dx = (b+1)a = \frac{1}{2}$$

Substituting for  $a$ , using the first equation, we get:

$$(b+1)\frac{2}{b+9} = \frac{1}{2}.$$

Solving, this gives us  $b = \frac{5}{3}$ . Substituting this into the first equation, we get  $a = \frac{3}{16}$ .

**Answer:**  $a = \underline{\frac{3}{16}}$        $b = \underline{\frac{5}{3}}$

- b. [2 points] Circle the one statement best supported by the equation

$$p(4.6) = 0.0225.$$

- i)  Anya will score between 4.5 and 4.7 points on about 0.45% of her turns.
- ii)  Anya will score 4.6 points on 2.25% of her turns.
- iii)  Anya will score 4.6 points on about 2.25% of her turns.
- iv)  Anya will score at most 4.6 points on about 2.25% of her turns.
- v)  Anya will score between 4.6 and 4.65 points on about 2.25% of her turns.
- vi)  Anya will score 0.0225 points on about 4.6% of her turns.
- vii)  Anya will score between 0 and 0.0225 points on about 0.1035% of her turns.