

7. [6 points] Determine whether the following series converges or diverges. Be sure to fully justify your answer, showing all work and indicating any theorems you use.

$$\sum_{n=1}^{\infty} \frac{(n-1)(\cos^2(n))}{7n^6 + 2n^4 + n}$$

Answer (Circle one):

Diverges

Converges

**Justification:**

*Solution:* Notice

$$\frac{(n-1)(\cos^2(n))}{7n^6 + 2n^4 + n} \leq \frac{n}{7n^6 + 2n^4 + n} < \frac{n}{7n^6} = \frac{1}{7n^5}.$$

The series  $\sum_{n=1}^{\infty} \frac{1}{7n^5}$  converges, as it is a  $p$ -series, with  $p = 5 > 1$ . So, by the (direct) comparison test,  $\sum_{n=1}^{\infty} \frac{(n-1)(\cos^2(n))}{7n^6 + 2n^4 + n}$  converges.