9. [9 points]

A small pond has murky water, and needs to be completely drained.

- A side view of the pond looks like the diagram at right.
- y measures the distance, in meters, above the bottom of the pond.
- The surface of the pond is at $y = 4$.
- The water must be pumped to a height **1 meter above the surface.**
- The cross-sections perpendicular to the y-axis are **circles.**
- The **radius** of the circular cross-section y meters above the bottom of the pond is $r(y)$ meters.
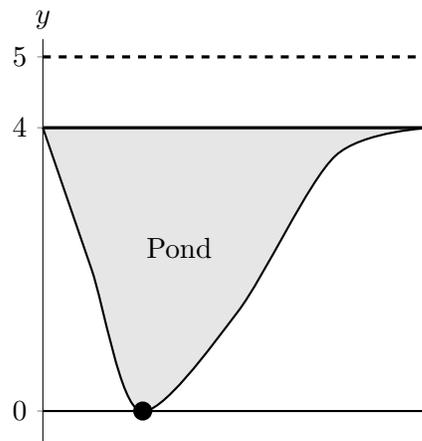
- The **density** of the murky water varies with y, and is given by $Q(y)$ kg/m^3.
- Note that the domain for both r and Q is $[0, 4]$.
- You may assume that acceleration due to gravity is $g = 9.8$ m/s^2.

Note that your answers below may include $r(y)$ and $Q(y)$.

a. [3 points] Write an expression that gives the approximate mass, in kilograms, of a slice of the murky water that is Δy m thick and at a height of y meters. Your expression should not involve any integrals.

Answer: Mass of slice $\approx$ $\underline{\qquad \pi r(y)^2 Q(y)\Delta y \text{ kg} \qquad}$

b. [3 points] Write an expression in terms of y that approximates the work, in joules, done in pumping a horizontal slice of murky water of thickness Δy at a height of y meters to 1 meter above the surface of the pond. Your expression should not involve any integrals.

Answer: Work $\approx$ $\underline{\qquad 9.8(5-y)\pi r(y)^2 Q(y)\Delta y \text{ Joules} \qquad}$

c. [3 points] Write an expression involving one or more integrals that gives the total work, in joules, to completely drain the pond by pumping all the water to 1 meter above the pond.

Answer: $\underline{\qquad \displaystyle\int_0^4 9.8(5-y)\pi r(y)^2 Q(y)\,dy \text{ Joules} \qquad}$