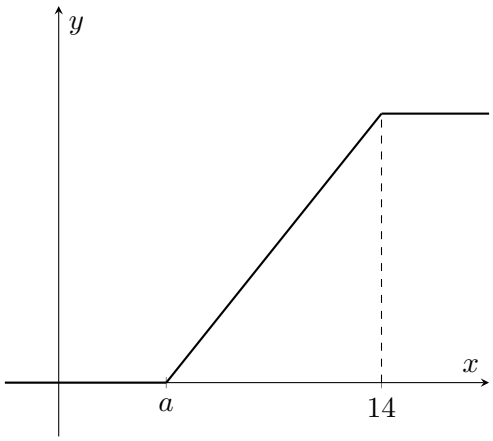
1. [8 points] The graph of a piecewise-linear cumulative distribution function $P(x)$ is given below. The function $P(x)$ is defined for all real numbers x and is constant for $x < a$ and for $x > 14$.



a. [3 points] The median value for x is 10. Use this fact, and the fact that $P(x)$ is a cumulative distribution function to find the value of a. No justification is needed but you may earn partial credit if you show your work.

Solution: Since the median for x is 10, we must have $P(10) = 0.5$.
Since $P(x)$ is a cdf, we must have $P(14) = 1$.
The slope of $P(x)$ between a and 14 is then $\frac{0.5}{4} = \frac{1}{8}$, and by using the equation of a straight line, we see that, between a and 14, $P(x) = \frac{1}{8}x - \frac{6}{8}$. Since $P(a) = 0$, we must have $a = 6$.

b. [5 points] Find a formula for a probability density function $p(x)$ which corresponds to $P(x)$. Make sure to define your formula for all values of x, using a piecewise-defined formula if necessary. You may give your answer in terms of a.

Solution: We must have $P'(x) = p(x)$ wherever it is defined. Using our formula from part a), we see that

$$p(x) = \begin{cases} 0 & \text{for } x < 6 \\ \frac{1}{8} & \text{for } 6 < x < 14 \\ 0 & \text{for } x > 14 \end{cases}$$