1. [8 points] The graph of a piecewise-linear cumulative distribution function \( P(x) \) is given below. The function \( P(x) \) is defined for all real numbers \( x \) and is constant for \( x < a \) and for \( x > 14 \).

\[
\begin{align*}
\text{y} \\
\text{a} \\
\text{14} \\
x
\end{align*}
\]

a. [3 points] The median value for \( x \) is 10. Use this fact, and the fact that \( P(x) \) is a cumulative distribution function to find the value of \( a \). No justification is needed but you may earn partial credit if you show your work.

**Solution:** Since the median for \( x \) is 10, we must have \( P(10) = 0.5 \).

Since \( P(x) \) is a cdf, we must have \( P(14) = 1 \).

The slope of \( P(x) \) between \( a \) and 14 is then \( \frac{0.5}{14 - a} = \frac{1}{8} \), and by using the equation of a straight line, we see that, between \( a \) and 14, \( P(x) = \frac{1}{8}x - \frac{6}{8} \). Since \( P(a) = 0 \), we must have \( a = 6 \).

b. [5 points] Find a formula for a probability density function \( p(x) \) which corresponds to \( P(x) \). Make sure to define your formula for all values of \( x \), using a piecewise-defined formula if necessary. You may give your answer in terms of \( a \).

**Solution:** We must have \( P'(x) = p(x) \) wherever it is defined. Using our formula from part a), we see that

\[
p(x) = \begin{cases} 
0 & \text{for } x < 6 \\
\frac{1}{8} & \text{for } 6 < x < 14 \\
0 & \text{for } x > 14 
\end{cases}
\]