2. [15 points]

a. [9 points] For each of the following sequences, defined for $n \geq 1$, state clearly whether the sequence is:

- increasing, decreasing, or neither.
- bounded or unbounded.
- convergent or divergent.

No justification is needed.

(i) $a_n = 2 - \cos(\pi n)$

Solution: neither, bounded, divergent

(ii) $b_n = \displaystyle\int_1^{n^2} \frac{1}{x}\, dx$

Solution: increasing, unbounded, divergent

(iii) $c_n = 13 - \displaystyle\sum_{k=0}^{n} \frac{1}{(1.1)^k}$

Solution: decreasing, bounded, convergent

b. [6 points] Let $\displaystyle\sum_{n=1}^{\infty} d_n$ be a geometric series, with $d_2 = 16$ and $d_5 = 2$. Determine, and clearly state, whether the series converges or diverges. If the series converges, find its sum.

Solution: Since the series is geometric, we know that $16x^3 = 2$, where x is the ratio in the series. Solving this gives $x = \frac{1}{2}$. This means the first term in the series is $a = 32$ (since $\frac{1}{2}a = 16$). Since the common ratio is $\frac{1}{2}$, the geometric series converges. Using this a and x, the sum is $32/(1 - 1/2) = 64$.