3. [10 points]

a. [6 points] Determine the **radius** of convergence for the following power series. Show all of your work. You do not need to find the interval of convergence.

$$\sum_{n=1}^{\infty}(-1)^n \frac{4^{n+1}}{n^{1/3}}(x-1)^n$$

Solution: We will use the ratio test. For $a_n = (-1)^n \frac{4^{n+1}}{n^{1/3}}(x-1)^n$, we have:

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{(-1)^{n+1}}{(-1)^n}\frac{4^{n+2}}{4^{n+1}}\frac{n^{1/3}}{(n+1)^{1/3}}\frac{(x-1)^{n+1}}{(x-1)^n}\right| \\
&= \lim_{n\to\infty} 4\frac{n^{1/3}}{(n+1)^{1/3}}|x-1| \\
&= 4|x-1|.
\end{aligned}$$

By the ratio test, the power series converges when $4|x-1| < 1$, i.e. $|x-1| < \frac{1}{4}$, and so the radius of convergence is $\frac{1}{4}$.

b. [4 points] Suppose the power series $\sum_{n=0}^{\infty} C_n(x-a)^n$ has radius of convergence 2, and that the series converges for $x = 4$ and diverges for $x = 6$. Which of the following could be the value of a? List **all** correct answers.

0 1 2 3 4 5 6

Solution: The series is centered at $x = a$ and has radius of convergence 2. Since the series converges at $x = 4$, we must have $2 \leq a \leq 6$. Since the series diverges at $x = 6$, we cannot have $4 < a < 8$. From the list, the only possible values of a are 2, 3, and 4.