4. [13 points] Rafael finds details of another of TimberCorp's logging operations, this time in a forest of redwoods which initially has 50,000 trees. TimberCorp plans to, at the start of each year, cut down 10% of the trees in the forest, and then over the course of the year replant k trees.

a. [5 points] Let R_n be the number of trees in the forest **at the end** of the nth year of the logging operation. Find expressions for R_1 and R_2. Your answers may involve k. **You do not need to simplify your answers**.

Solution:

$$R_1 = (0.9)50000 + k$$

$$R_2 = (0.9)^2(50000) + 0.9k + k$$

b. [5 points] Find a **closed form** expression for R_n. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. **You do not need to simplify your closed form answer.**

Solution:

$$\begin{aligned} R_n &= (0.9)^n(50000) + k + 0.9k + (0.9)^2k + \ldots + (0.9)^n k \\ &= (0.9)^n(50000) + k\left(\tfrac{1-(0.9)^n}{1-0.9}\right) \\ &= (0.9)^n(50000) + 10k\left(1 - (0.9)^n\right) \end{aligned}$$

where we used the formula for the sum of a (finite) geometric series

c. [3 points] Rafael wants the number of trees in the forest at the end of a year to tend towards 70,000 in the long run (i.e. after many many years). What value should he choose for k to ensure this happens?

Solution: As $n \to \infty$, $R_n \to 0 + 10k(1 - 0) = 10k$, and so R_n will tend toward 70,000 if $10k = 70000$, i.e. $k = 7000$.