5. [14 points]

a. [7 points] Determine whether the following improper integral converges or diverges. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\int_0^1 \frac{3}{x+x^{1/2}}\,dx$$

Solution: On the interval $0 \le x \le 1$, we have $\frac{3}{x+x^{1/2}} \le \frac{3}{x^{1/2}}$, and $\int_0^1 \frac{3}{x^{1/2}}\,dx$ converges by the p-test with $p = \frac{1}{2}$. Therefore, by the (Direct) Comparison Test, $\int_0^1 \frac{3}{x+x^{1/2}}\,dx$ converges.

b. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_1^\infty \frac{2+2e^x}{(x+e^x)^{3/2}}\,dx$$

Solution:

$$\int_1^\infty \frac{2+2e^x}{(x+e^x)^{3/2}}\,dx = \lim_{b\to\infty}\int_1^b \frac{2+2e^x}{(x+e^x)^{3/2}}\,dx.$$

Now, using the substitution $w = x + e^x$, we see

$$\int_1^b \frac{2+2e^x}{(x+e^x)^{3/2}}\,dx = \int_{1+e}^{b+e^b} \frac{2}{w^{3/2}}\,dw = -\frac{4}{w^{1/2}}\Big|_{1+e}^{b+e^b} = \frac{4}{(1+e)^{1/2}} - \frac{4}{(b+e^b)^{1/2}}.$$

Therefore

$$\int_1^\infty \frac{2+2e^x}{(x+e^x)^{3/2}}\,dx = \lim_{b\to\infty}\left(\frac{4}{(1+e)^{1/2}} - \frac{4}{(b+e^b)^{1/2}}\right) = \frac{4}{(1+e)^{1/2}}.$$