8. [14 points] Determine whether the following series converge or diverge. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

a. [7 points]

$$\sum_{n=1}^{\infty} \frac{3-\cos(n)}{6n-n^{1/2}}$$

Solution: We have, for $n \geq 1$,

$$\frac{3-\cos(n)}{6n-n^{1/2}} \geq \frac{2}{6n-n^{1/2}} \geq \frac{2}{6n} = \frac{1}{3n}$$

and $\sum_{n=1}^{\infty} \frac{1}{3n}$ diverges by the p-test (with $p = 1$).

Therefore, by the Comparison Test, $\sum_{n=1}^{\infty} \frac{3-\cos(n)}{6n-n^{1/2}}$ diverges.

b. [7 points]

$$\sum_{n=2}^{\infty} \frac{3}{n(\ln(n))^2}$$

Solution: Use the integral test, where $f(x) = \frac{3}{x(\ln(x))^2}$ is positive and decreasing.

$$\begin{aligned}\int_2^{\infty} \frac{3}{x(\ln(x))^2} dx &= \lim_{b\to\infty} \int_2^b \frac{3}{x(\ln(x))^2} dx \\ &= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{3}{u^2} du \quad \text{after using the substitution } u = \ln(x) \\ &= \int_{\ln 2}^{\infty} \frac{3}{u^2} du,\end{aligned}$$

which converges by the p-test with $p = 2$. Therefore, by the integral test, the original series converges as well.